International Journal of Approximate Reasoning 94 (2018) 43–59

Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

www.elsevier.com/locate/ijar

Labeled fuzzy approximations based on bisimulations

Yibin Du [a,b,*], Ping Zhu [c]

[a] Automation school, Beijing University of Posts and Telecommunications, Beijing 100876, China
[b] University of Foreign Languages, Luoyang 471003, Henan, China
[c] School of Science, Beijing University of Posts and Telecommunications, Beijing 100876, China

ARTICLE INFO

Article history:
Received 4 August 2016
Received in revised form 13 November 2017
Accepted 9 January 2018

Keywords:
Rough set
Fuzzy set
Labeled fuzzy approximation space
Bisimulation
Bisimilarity
Simulation

ABSTRACT

This paper focuses on the labeled fuzzy approximation space, which is considered as a relational structure consisting of a nonempty universal set and some fuzzy relations. To deduce knowledge hidden in the labeled fuzzy approximation space, based on the notion of bisimulations, lower and upper fuzzy rough approximation operators are constructed. Then basic properties of the fuzzy rough approximation operators are investigated. When the largest bisimulation is a trivial identity relation in some cases, the concept of simulations is proposed. Moreover, the lower and upper fuzzy rough relation approximation operators are first proposed and properties of the new operators are examined. Finally, the relationships between two kinds of approximations are discussed.

© 2018 Elsevier Inc. All rights reserved.

1. Introduction

Rough set theory was originally proposed by Pawlak in 1982 to deal with uncertain, incomplete and vague data in information systems [19,20]. It has been successfully applied in pattern recognition, data mining, automated knowledge acquisition and many other fields [21–23,16,34,35,39]. Pawlak rough set theory is constructed on equivalence relations. However, equivalence relations impose limitations when dealing with real-valued data sets [13]. Fuzzy set theory proposed in 1965 by Zadeh [36] is very useful to overcome these limitations, as it can deal effectively with vague concepts and graded indiscernibility. To describe fuzzy concepts, Dubois and Prade [8,9] extended the basic idea of rough sets to a new model called fuzzy rough sets, which were further investigated with different fuzzy logic operations and binary fuzzy relations in [1,17,25,29–31,33]. Recently, D'eer et al. [10] considered these different proposals from the view of the implicator–conjunctor-based fuzzy rough set model. Wang [28] pointed out that neither implicator–conjunctor-based fuzzy rough set model nor fuzzy variable precision rough set model [38] satisfies (ID) property without the reflexivity of fuzzy relation. It should be noted that most of different fuzzy rough set models are based on fuzzy approximation spaces which are comprised of a nonempty universal set and a fuzzy relation or a crisp relation. Since every crisp relation can be seen as a special case of a fuzzy relation, we think that a fuzzy approximation space consists of a nonempty universal set and a fuzzy relation. The fuzzy relation is used to model indiscernibility. Using the lower and upper fuzzy rough approximations induced by the fuzzy relation, the knowledge hidden in fuzzy approximation spaces can be discovered and expressed.

In essence, the aforementioned fuzzy rough approximations based on fuzzy relations are only dependent on "one step" information of the underlying fuzzy relations. It extends the meaning of "one step" information of binary relations [37]. Zhu et al. [37] mean that the ordered pair of the starting and end points of the step belongs to the relation by "one step" in a

* Corresponding author at: Automation school, Beijing University of Posts and Telecommunications, Beijing 100876, China.
E-mail addresses: duyibindu@163.com (Y. Du), pzhubupt@gmail.com (P. Zhu).

https://doi.org/10.1016/j.ijar.2018.01.002
0888-613X/© 2018 Elsevier Inc. All rights reserved.

binary relation. In the fuzzy environment, for a fuzzy relation $R \in \mathcal{F}(S \times S)$ and any $(x, y) \in S \times S$, we say that the degree $R(x, y)$ of R represents "one step" information of R. For instance, for each x the fuzzy set R_x of S where $R_x(y) = R(x, y)$ for any y means all "one step" information from x. In general, the fuzzy rough approximations are based on the fuzzy set R_x. For example, the lower and upper fuzzy rough approximations defined respectively by $\underline{R}(A)(x) = \wedge\{A(y) \vee (1 - R_x(y)) \mid y \in S\}$ for any $x \in S$ and $\overline{R}(A)(x) = \vee\{A(y) \wedge R_x(y) \mid y \in S\}$ for any $x \in S$ are typical ones which are based on the fuzzy set R_x [30]. This implies that the indiscernibility is characterized by a "one step" information of the underlying fuzzy relation. Now that "one step" information is useful for fuzzy rough approximations, a natural question is whether "multi-step" information can characterize indiscernibility. The question motivates us to write this paper.

Fortunately, there is an elegant concept called bisimulation [18,24], that reflects "multi-step" information. It is considered as one of the most important contributions of concurrency theory to computer science [12,6,7,26,2,27,5,4]. Bisimulation expresses that one process simulates the other and vice versa when two processes can behave in the same way. It is also exploited to reduce the state space. Recently, based on a binary relation on the state space of a fuzzy system, Cao et al. [4] investigated bisimulation for fuzzy transition systems which are characterized by a set of states, a set of labels, a fuzzy transition function and the initial state.

In this paper, we begin with fuzzy transition systems (FTSs) introduced by Cao et al. [4]. Then we propose the lower and upper fuzzy rough approximations based on bisimulation. In fact, an FTS is called a labeled fuzzy approximation space if the initial state is omitted. Furthermore because one can identify any fuzzy transition function with a fuzzy relation, a labeled fuzzy approximation space is considered as a relational structure which consists of a nonempty universal set and some fuzzy relations. It is easy to see that a fuzzy approximation space just has one fuzzy relation, but a labeled fuzzy approximation space has more than one fuzzy relations. The fuzzy relations can be understood as the knowledge that has been acquired. As an equivalence relation is used to model indiscernibility in the original model of Pawlak, a question to ask is how to collect the knowledge to model indiscernibility in a labeled fuzzy approximation space. The following question to ask is how to deduce knowledge hidden in a labeled fuzzy approximation space by the indiscernibility. They are clearly interesting mathematical questions, and in this paper we answer them affirmatively.

The rest of this paper is structured as follows: In Section 2, related notions and results of fuzzy sets, fuzzy relations, fuzzy approximation operators, and bisimulations are briefly introduced. In Section 3, based on the bisimulations in labeled fuzzy approximation spaces, the lower and upper fuzzy rough approximation operators are proposed and their properties are discussed. Furthermore, the concept of simulations is defined when the largest bisimulation is a trivial identity relation in some cases. Then using the simulations in labeled fuzzy approximation spaces, the lower and upper fuzzy rough approximation operators are given. In Section 4, the lower and upper fuzzy rough relation approximation operators are first proposed and properties of the new operators are examined. The relationships between two kinds of approximations are investigated in Section 5. The paper is then concluded in Section 6.

2. Preliminaries

In this section, we introduce some basic concepts and results of fuzzy sets, fuzzy relations, fuzzy approximation operators, and bisimulations, most of which are from [4,11,8,30,14,15,32].

2.1. Fuzzy set theory

Let S be a universal set and $A(x)$ be a function from S to [0,1]. Such a function is called a membership function, which is a generalization of the characteristic function associated with a subset of S. A fuzzy set A of S is characterized by a membership function $A(x)$ with the values of $A(x)$ at x representing the degree of membership of x in A. It is clear that A is completely determined by the set of tuples $\{(x, A(x)) \mid x \in S\}$. The set of all fuzzy sets of S is denoted by $\mathcal{F}(S)$. The support of a fuzzy set A is a set defined as $supp(A) = \{x \in S \mid A(x) > 0\}$. If $supp(A) = \{x_1, x_2, \cdots, x_n\}$ is a finite set, then we often use Zadeh's notation

$$A = A(x_1)/x_1 + A(x_2)/x_2 + \cdots + A(x_n)/x_n$$

where the term $A(x_i)/x_i$ and $i = 1, 2, \cdots, n$, signifies that $A(x_i)$ is the degree of membership of x_i in A and the plus sign represents the union.

Let A and B be fuzzy sets of S. We say that A is contained in B if $A(x) \le B(x)$ for all $x \in S$, which is denoted by $A \subseteq B$. A and B are said to be equal, which is denoted by $A = B$, if $A \subseteq B$ and $B \subseteq A$. The intersection of A and B is defined as $(A \cap B)(x) = min\{A(x), B(x)\} = A(x) \wedge B(x)$, for all $x \in S$. The union of A and B is defined as $(A \cup B)(x) = max\{A(x), B(x)\} = A(x) \vee B(x)$, for all $x \in S$. The complement of a fuzzy set A is defined as $A^c(x) = 1 - A(x)$, for all $x \in S$. De Morgan's laws of A and B are $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$.

For convenience, we adopt the following notations given by Cao and Chen in [4]. For any family $\lambda_i, i \in I$, of elements of [0,1], we write $\vee\{\lambda_i \mid i \in I\}$ for the supremum of $\{\lambda_i \mid i \in I\}$, and $\wedge\{\lambda_i \mid i \in I\}$ for the infimun. In particular, if I is finite, then $\vee\{\lambda_i \mid i \in I\}$ and $\wedge\{\lambda_i \mid i \in I\}$ are the greatest element and the least element of $\{\lambda_i \mid i \in I\}$, respectively. For any $\mu \in \mathcal{F}(S)$ and $U \subseteq S$, the notation $\mu(U)$ stands for $\vee\{\mu(x) \mid x \in U\}$.

Download English Version:

https://daneshyari.com/en/article/6858823

Download Persian Version:

https://daneshyari.com/article/6858823

Daneshyari.com