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Modelling incomplete information in Boolean games using possibilistic logic [☆]

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ABSTRACT

Boolean games (BGs) are a framework for specifying strategic games in which the utility of an agent is determined based on the satisfaction of goals in propositional logic. The majority of existing work on BGs relies on the often unrealistic assumption that agents have perfect knowledge of each other's preferences. In this paper, we show how this issue can be addressed in a natural way, by replacing the use of classical logic for expressing agents' goals by possibilistic logic. We consider two such settings. In the first setting, possibilistic logic is used to encode knowledge about other agents' goals with different levels of certainty. In the second setting, which is based on generalized possibilistic logic, certainty levels are instead used to compactly encode priorities, while incompleteness is modelled in a binary way, similar as in epistemic modal logics. In both cases we introduce natural solution concepts, motivated by Schelling's theory on focal points: a certain pure Nash equilibrium (PNE) is preferred over another one due to the fact that all agents *know* it to be a PNE. Alternatively, an outcome might be preferred when all agents *consider it possible* of being a PNE. We prove that the associated computational complexity of these solution concepts does not increase compared to PNEs in Boolean games with complete information. Finally, to illustrate the practical relevance, we consider an application to negotiation, among others showing how knowledgeable agents can obtain a more desirable outcome than others.

1. Introduction

Game-theoretic frameworks often rely on the somewhat artificial assumption that agents are fully aware of each other's goals. In strategic settings, agents might deliberately conceal such information, or might not have had a chance to exchange it. Even if an agent knows its opponents well, it may not be fully certain about what exact goals the other agents are pursuing. For instance, suppose Alice and Bob are married and plan a night out. The options are going to the theatre and attending a sports game. Even if we assume that Alice and Bob have been married for an eternity and know each other *inside out*, they are not mind readers: it is still possible that Alice is not entirely sure whether Bob really prefers to join her

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to the theatre over attending the sports game alone. Games with incomplete information [1,2] allow us to explicitly model the limitations of agents' knowledge about the preferences of the others. While this topic has been extensively researched for normal form games, Boolean games with incomplete information have hardly received any attention; see Section 2 for a discussion.

In this paper, we study the use of possibilistic logic [3] to model Boolean games with incomplete information. Possibilistic logic has the advantage of staying close to classical logic, while offering us more flexibility, and it can be naturally used to model (partial) ignorance [4]. Specifically, we study two different settings which differ in how the necessity degrees from the possibilistic logic theories are interpreted. In the first setting, we consider the usual interpretation of necessity degrees as certainty degrees. It uses possibilistic knowledge bases to encode necessary and sufficient conditions for the satisfaction of other agents subgoals. For instance, Alice can encode that she is absolutely certain that Bob reaches his highest utility when they both attend the sports game, while she is less certain that Bob reaches his highest utility when they both go to the theatre. We prove that this framework at the semantic level corresponds to a possibility distribution over all possible games.

In the second approach, necessity degrees are instead used to model preference. To keep the ability to model incomplete information, for this setting we switch from standard possibilistic logic to generalized possibilistic logic (GPL) [4]. In particular, each GPL theory semantically corresponds to a set of possibility distributions. In our context, these possibility distributions are interpreted as the utility functions that, according to a given agent, may correspond to the actual utility function of some other agent.

For both frameworks of Boolean games with incomplete information, we propose intuitive solution concepts, reflecting whether agents know or consider it possible that a certain outcome is a pure Nash equilibrium (PNE). We prove that the computational complexity of the associated decision problems does not increase compared to PNEs in Boolean games with complete information. To illustrate how these solution concepts could be useful, we briefly discuss an application to negotiation in Boolean games with incomplete information.

The remainder of this paper is structured as follows. We first discuss related work in Section 2 and give some background on Boolean games and possibilistic logic in Section 3. Next we introduce our two approaches in Section 4 and Section 5. Finally, we present the application to negotiation in Section 6. Finally note that this paper is an extension of our work in [5] (Section 4) and [6] (Sections 5 and 6). In addition to providing more detailed explanations and proofs, we have extended the framework from [5] with joint constraints and have added complexity results to the framework from [6].

2. Related work

Although uncertainty has been studied extensively in the context of game theory (see e.g. [1]), the literature on Boolean games with incomplete information is currently limited. Note that we are only concerned with epistemic uncertainty in this paper (e.g. we do not consider stochastic actions, whose outcome cannot be predicted with certainty). To the best of our knowledge, stochastic uncertainty has not yet been studied in the context of Boolean games. Moreover, we are not aware of any existing approaches for modelling uncertainty w.r.t. the goals of other agents, in the context of Boolean games, although uncertain Boolean games have been studied for other purposes. For example, in [7] uncertain Boolean games are modelled by introducing a set of environment variables which are outside the control of any agent. Each agent has some (possibly incorrect) belief about the value of the environment variables. The focus of [7] is on manipulating Boolean games by making announcements about the true value of some environment variables, in order to create a stable solution if there were none without the announcements. In [8] uncertainty is modelled by extending the framework of Boolean games with a set of observable action variables for every agent, i.e. every agent can only observe the values assigned to a particular subset of action variables. As a result, agents are not able to distinguish between some outcomes, if these profiles only differ in action variables that are not observable to that agent. Three notions of verifiable equilibria are investigated, capturing respectively outcomes for which all agents know that they *might be* pure Nash equilibria (PNEs), outcomes for which all agents know that they *are* PNEs and outcomes for which it is common knowledge that they are PNEs, i.e. all agents know that they are PNEs and all agents know that all agents know that they are PNEs etc. The same authors have extended this framework to epistemic Boolean games [9], in which the logical language for describing goals is broadened to a multi-agent epistemic modal logic. Note, however, that agents are still fully aware of each other's goals in this framework, i.e. [9] considers agents whose goal is to obtain a particular epistemic state. For instance, *I not only want my husband to pick up our baby, I also want to know he is picking up our baby*.

In contrast, we study Boolean games with incomplete information, considering agents which have their own beliefs about the goals of other agents. Although probability theory is often used to model uncertainty in game theory [1], a possibilistic logic approach provides a simple and elegant mechanism for modelling partial ignorance, which is closely related to the notion of epistemic entrenchment [10]. Being based on ranking formulas (at the syntactic level) or ranking possible worlds (at the semantic level), possibilistic logic has the advantage of staying close to classical logic. As a result, we can introduce methods for solving possibilistic Boolean games that are closely similar to methods for solving standard Boolean games.

Within the broader context of game theory, several authors have looked at qualitative ways of modelling epistemic uncertainty. A common approach is to model the beliefs of an agent a about another agent b as a set of pairs (s, t), where s is a strategy and t is a so-called type (where types are used to model beliefs about beliefs in a hierarchical way). Such a belief structure is sometimes called *possibilistic* in the game theory literature (e.g. [11,12]). However, it should be