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Evidential box particle filter using belief function theory

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ABSTRACT

A box particle filtering algorithm for nonlinear state estimation based on belief function theory and interval analysis is presented. The system under consideration is subject to bounded process noises and Gaussian multivariate measurement errors. The mean and the covariance matrix of Gaussian random variables are considered bounded due to modeling errors. The belief function theory is a means to represent this type of uncertainty using a mass function whose focal sets are intervals. The proposed algorithm applies interval analysis and constraint satisfaction techniques. Two nonlinear examples show the efficiency of the proposed approach compared to the original box particle filter.

1. Introduction

Dynamic state estimation is an important task in many engineering applications. It is useful to design control laws or monitor system performance. A common issue in the design of filters is to find a representation of uncertainties that is consistent with state space models and system perturbations. Two categories of state estimation approaches can be distinguished according to the modeling of uncertainties.

The first group of methods is based on a statistical description of uncertainties, i.e., it assumes uncertainties to be realizations of random variables with known probability distributions. The main idea of stochastic filters is to form an approximation for the a posteriori state probability distribution given the measurements on the system. When the system model is linear with Gaussian noises, the *Kalman filter* [17] computes new estimates of the state of interest by simply taking a weighted average of the model-based prediction, based on the previous estimate, and a new observation. However, models representing real systems are usually nonlinear with uncertainties such as uniform noises or bounded parameter uncertainties. The *Extended Kalman filter* (EKF) is a means to deal with nonlinear models: the state mean and variance are approximated by propagating Gaussian random variables through the linearized model [23]. When the system is highly nonlinear with non-Gaussian noises, the state estimation is better handled by Sequential Monte Carlo methods, so-called *Particle filters* (PF) [14]. A PF is expected to provide an approximation for the a posteriori state distribution in partially observable controllable Markov chain. However, a drawback of PF is related to the computational cost due to the large number of particles required if there is a high degree of imprecision. Several algorithms have been developed (for instance [29], [31]) to reduce the number of particles required to obtain an appropriate representation of the a posteriori density, thus reduce the associated computational time.

In the second group of methods, uncertainties are assumed to be bounded by known compact sets, such as ellipsoids, zonotopes or boxes, i.e., no statistical assumption is then required. This group of methods, known as set-membership

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approaches, attempts to provide sets containing all state values consistent both with the uncertain model and the measurements. Interval analysis that has been introduced in [19] and overviewed in [16] is a tool for set-membership state estimation. In particular, the combination of PF and interval analysis, called the *box particle filter* (BPF) ([1], [4]) allows one to deal with bounded uncertainties and to reduce the number of particles. The major difficulty of this approach is to bound the uncertainties. Indeed, the overestimation of the bounds leads to impreciseness of state estimation, and the method becomes quite pessimistic. On the contrary, if the bounds are too tight, the data may become inconsistent with the system model and the measurements.

A useful tool which allows one to represent incomplete statistical knowledge is belief function theory. This theory, also known as evidence theory or Dempster–Shafer (DS) theory, was first introduced in the context of statistical inference [6]. The theory was later formalized as a theory of evidence in [22] and has been developed by Smets under the name transferable belief model (see for instance, [24], [25], [26], [27]). The belief function theory can be viewed as a generalization of Bayesian probability calculus and the set-based formalism since it allows to assign belief masses to subsets of the hypothesis space [21], not only to single elements. From a given mass function, a probability interval containing a set of probability measures is defined by a couple of *belief-plausibility* measures. A special application of belief function theory is the characterization of a system whose knowledge is represented by a *pignistic probability* function [26]. In this case, an approach based on the construction of the least committed basic belief function from an univariate and unimodal probability density is proposed in [26]. The explicit expression of the least committed belief function induced by a n-dimensional Gaussian probability density was derived in [5]. This approach was applied to model based target classification in [2], [5].

The belief function theory has been little used in the context of filtering. Yet, a particle filter is presented within the Dempster–Shafer framework in [21] in which the knowledge about the current state, state transitions and likelihoods are all represented as mass functions. In [20], the authors present a box particle filter using mass functions with a *finite* number of focal sets under the form of boxes to express the information of the states and noises. Focal sets of these mass functions are propagated through system equations by interval analysis and constraint satisfaction techniques [16]. Nevertheless, this method cannot be used for mass functions of Gaussian densities due to their infinite number of focal elements.

In some cases of Gaussian densities, only a confidence region of statistical model parameters can be determined due to modeling errors. Motivated by this problem, this paper proposes a box particle filtering algorithm for nonlinear systems in which the process noises are bounded, but the measurement errors are modeled by a Gaussian multivariate distribution whose mean and covariance matrix are considered as intervals. A continuous mass function with focal boxes representing this type of uncertain Gaussian density is constructed using the *least commitment principle* of the belief function theory. We use the probability intervals of box particles given by the proposed mass function to detect when the box particles are too pessimistic in order to redistribute the set of box particles. This approach can be considered as a robust solution to the filtering problem with stochastic disturbances and bounded parameter uncertainty.

The paper is organized as follows. After formulating the problem in Section 2, Section 3 presents the basic tools of interval analysis. The main concepts of belief function theory are recalled and applied to an uncertain Gaussian density in Section 4. The original box particle filter is summarized in Section 5. Then the proposed algorithm is derived in Section 6, followed by two numerical examples in Section 7. The final section concludes and discusses the obtained results.

2. Problem formulation

Consider the following nonlinear system

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{w}_k), \\ \mathbf{y}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k, \end{cases} \tag{1}$$

where $\mathbf{x}_k \in \mathbb{R}^{n_x}$, $\mathbf{u}_k \in \mathbb{R}^{n_u}$ and $\mathbf{y}_k \in \mathbb{R}^{n_y}$ are the state, input and measurement vectors. The process noise $\mathbf{w}_k \in \mathbb{R}^{n_x}$ is assumed to be bounded by a box $[\mathbf{w}_k]$. The measurement errors $\mathbf{v}_k \in \mathbb{R}^{n_y}$ are represented by a Gaussian multivariate distribution $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are bounded: $\boldsymbol{\mu} \in [\boldsymbol{\mu}]$, $\boldsymbol{\Sigma} \in [\boldsymbol{\Sigma}]$. The functions $\mathbf{f} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_x}$, and $\mathbf{h} : \mathbb{R}^{n_x} \to \mathbb{R}^{n_y}$ may be nonlinear functions. The initial state $\mathbf{x}_0$ belongs to a box $[\mathbf{x}_0]$.

Within the Bayesian framework, the state $\mathbf{x}_k$ at any time step k is estimated from the a posteriori density $p(\mathbf{x}_k \mid \mathbf{y}_0, \ldots, \mathbf{y}_k)$. Considering the assumptions presented above, these conditional probabilities cannot be determined by the classical probability theory due to the imprecision of the measurement error density. An alternative tool to manage these uncertainties is the *belief function theory*. It represents an incomplete information by a mass function and provides a probability interval for any subset of the state space. In this paper, a box particle filter is proposed using belief function theory and interval analysis to deal with mixed stochastic and bounded uncertainties.

Interval analysis and belief function theory are presented in two next sections.

3. Interval analysis

This section introduces basic notions of interval analysis, mainly taken from [19] and [16], that are useful to deal with bounded uncertainties. Constraint satisfaction problem using interval analysis is also presented.