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On multivariate asymmetric dependence using multivariate skew-normal copula-based regression

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ABSTRACT

In this paper we propose a procedure to study the asymmetric dependence of the multivariate data. The proposed procedure comprises methodologies that have not been considered in the analysis of multivariate asymmetric dependence. We first utilize the asymmetric multivariate copula-based regression to capture the asymmetric dependence among multiple variables. We then introduce the multiple asymmetric dependence measure to quantify the asymmetry in the predictive power of the tentative predictors for a tentative response variable. We demonstrate the proposed methods using a class of asymmetric multivariate skew normal copulas. An application example on the asymmetric comovements of financial assets illustrates the benefits of the proposed methods.

1. Introduction

There is a great need for modeling asymmetric dependence structure in various research contexts such as finances [3,5,20,54,56], gene networks [29], developmental research on attention deficit hyperactivity disorder [36] and aggression in adolescence [50]. By asymmetric dependence, we mean that the associations/interactions are not always identical among all variables involved and thus the variables influence each other with different magnitude.

Modeling dependence with the copulas and the copula-based regression has recently drawn attention in literature [9,14,19,28,29,43,45,46,53]. The asymmetric (non-exchangeable/radial asymmetric) copulas are flexible in describing asymmetric dependence stemming from the joint behavior of the variables separated from their marginal behaviors. The regression approach enables quantification of the degree of asymmetric dependence captured by asymmetric copulas. However, the main limitation of the previous researches is their focus on the bivariate and/or symmetric case.

Considerable efforts have been put into developing multivariate copulas describing asymmetric dependence structures. The hierarchical Archimedean copulas [33], constructed from the idea of the compositions of simple Archimedean copulas, is designed for non-exchangeable dependency structures. The vine copulas [2,6,21,44] decompose a multivariate distribution into a product of conditional bivariate copulas and determine the dependence structure by a cascade of bivariate copulas. The Liouville copulas [34] are proposed as a non-exchangeable generalization of the Archimedean copulas. The class of multivariate skew normal copulas, derived from the multivariate skew normal distribution [4], has two sets of parameters capturing asymmetric dependence in the data [51].

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In this paper we propose a procedure to study the asymmetric dependence of the multivariate data. The proposed procedure utilizes methodologies that have not been considered in the analysis of multivariate asymmetric dependence. Our contribution is twofold: First, we propose using a flexible class of multivariate skew normal copula-based regression to capture the asymmetric dependence in the multivariate data. Second, we propose a measure of multivariate asymmetric dependence designed to take account of the asymmetry in the predictive power of a set of tentative predictors for a tentative response variable. The proposed measure, extending the results of the generalized measures of correlation in [57] from a bivariate case to a multivariate case, is the first such for evaluating the asymmetric/nonlinear dependence among multiple variables via the copula-based regression.

The rest of the article is structured as follows. Section 2 briefly reviews a class of multivariate skew normal copulas proposed in [51] and discusses its asymmetry properties (non-exchangeability and radial asymmetry). Section 3 proposes the multivariate skew normal copula-based regression and its semiparametric estimation. In Section 4, we propose a measure of multivariate asymmetric dependence to quantify the degree of asymmetric dependence captured by copula-based regression and investigate its theoretical properties. We also discuss the estimation of the proposed measure. Section 6 illustrates the proposed methods with a real data example on the asymmetric comovements of financial assets. We end this article with a discussion in Section 6.

2. Multivariate skew normal copula

In this section, we briefly review the definition of multivariate copulas, its symmetry properties (exchangeability and radial symmetry) and the multivariate skew normal copulas proposed in [51]. For a detailed overview of copula theory, see [13,23].

2.1. Review on multivariate copula

Denote $\boldsymbol{X} = (X_0, X_1, \ldots, X_k)^\top$ on $\mathbb{R}^{k+1}$ be a $(k+1)$-dimensional random vector with the joint cumulative distribution function (CDF) $H(\mathbf{x}) = P(X_0 \le x_0, X_1 \le x_1, \ldots, X_k \le x_k)$ and marginal CDFs $F_i(x_i)$, $i = 0, 1, \ldots, k$. For the random vector $\boldsymbol{X}$ with continuous margins $F_i(x_i)$, the Sklar's theorem [42] states that there exists a unique copula C such that

$$H(\mathbf{x}) = C(F_0(x_0), F_1(x_1), \ldots, F_k(x_k)), \tag{2.1}$$

where C is a multivariate distribution function with uniform marginal distributions, that is, the function from $[0,1]^{k+1}$ to $[0,1]$ defined by

$$\begin{aligned} C(\mathbf{u}) &\equiv P(U_0 \le u_0, U_1 \le u_1, \ldots, U_k \le u_k) \\ &= H(F_0^{-1}(u_0), F_1^{-1}(u_1), \ldots, F_k^{-1}(u_k)), \end{aligned} \tag{2.2}$$

$\mathbf{U} = (U_0, U_1, \ldots, U_k)^\top$ with $U_i = F_i(X_i)$, and $F_i^{-1}(u_i) = \inf\{x : F_i(x) \ge u_i\}$ for $i = 0, 1, \ldots, k$. Note that a copula C in Eq. (2.2) allows us to separate the multivariate dependence between multiple variables from the univariate marginal distributions, and it contains information about the dependence structure of $H(\mathbf{x})$ on a quantile scale.

For an absolutely continuous copula C, the copula density is defined as

$$c(\mathbf{u}) = \frac{\partial^{k+1} C(u_0, u_1, \ldots, u_k)}{\partial u_0 \partial u_1 \cdots \partial u_k} = \frac{h(F_0^{-1}(u_0), F_1^{-1}(u_1), \ldots, F_k^{-1}(u_k))}{f_0(F_0^{-1}(u_0)) f_1(F_1^{-1}(u_1)) \ldots f_k(F_k^{-1}(u_k))},$$

where $h(\mathbf{x})$ is the joint density of $\boldsymbol{X}$ and $f_i(x_i)$ are the marginal density of X_i.

The exchangeability and the radial symmetry are two types of symmetry commonly imposed on the copula. The definitions of exchangeable and radially symmetric copulas are given below:

Definition 2.1. A $(k+1)$-copula C is exchangeable if it is the distribution function of a $(k+1)$-dimensional exchangeable uniform random vector $\boldsymbol{U} = (U_0, U_1, \ldots, U_k)^\top$ satisfying $C(u_0, u_1, \ldots, u_k) = C(u_{\sigma(0)}, u_{\sigma(1)}, \ldots, u_{\sigma(k)})$ for all $\sigma \in \Gamma$, where Γ denotes the set of all permutations on the set $\{0, 1, \ldots, k\}$.

Note that a $(k+1)$-dimensional continuous random vector $\boldsymbol{X}$ is exchangeable if and only if the marginal CDFs are identical and the copula C is exchangeable.

Definition 2.2. A $(k+1)$-copula C is radially symmetric if $\boldsymbol{U}$ has the same distribution as the random vector $\mathbf{1} - \boldsymbol{U}$ where $\mathbf{1} - \boldsymbol{U} = (1 - U_0, \ldots, 1 - U_k)^\top$.

Note that a $(k+1)$-dimensional random vector $\boldsymbol{X}$ is radially symmetric about $\boldsymbol{a} = (a_0, \ldots, a_k)$ if and only if X_i is marginal radially symmetric about a_i (i.e., the distribution functions of $X_i - a_i$ and $X_i + a_i$ are same) and the corresponding copula is radially symmetric.

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