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Comparison of reduction in formal decision contexts

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ABSTRACT

In formal concept analysis, many reduction methods have recently been proposed for formal decision contexts, and each of them was to reduce formal decision contexts with a particular purpose. However, little attention has been paid to the comparison of their differences from various aspects. In fact, this problem is very important because it can provide evidence to select an appropriate reduction method for a given specific case. To address this problem, our study mainly focuses on clarifying the relationship among the existing reduction methods in formal decision contexts. Firstly, we give a rule-based review of the existing reduction methods, revealing the type of rules that each of them can preserve. Secondly, we analyze the relationship among the consistencies introduced by the existing reduction methods. More specifically, Wei's first consistency (see [39]) is stronger than others, while her second one is weaker than the remainder except Wu's consistency (see [43]). Finally, we make a comparison of the existing reductions, concluding that Li's reduction (see [14]) maintaining the non-redundant decision rules of a formal decision context is coarser than others. The results obtained in this paper are beneficial for users to select an appropriate reduction method for meeting their requirements.

1. Introduction

Formal concept analysis (FCA) was proposed by Wille [41] to restructure lattice theory based on hierarchies of formal concepts. To be more specific, formal concepts together with a predefined subconcept–superconcept relation form a complete lattice, called concept lattice [8]. After 34 years of development, FCA has been successfully applied in many fields [5,7,31,42,44,51].

However, with the deepening of the research on FCA, many scholars have found that the constraints on Wille's concept lattice are too restrictive for many applications. In order to loosen these constraints, several types of generalized concept lattices were developed such as power concept lattice [10], rough concept lattice [12], three-way concept lattice [27], AFS concept lattice [35], object-oriented concept lattice [45,46] and approximate concept lattice [16]. Moreover, some studies have also been made on the relationship among the generalized concept lattices [22,37]. In addition, how to reduce these generalized concept lattices was discussed from the perspectives of matrix factorization [2], fuzzy K-means clustering [3], rough set [20], covering-based rough set [18], and lattice isomorphism [19,24,50]. Besides, mining implications or association rules based on the generalized concept lattices has also attracted much attention in recent years [1,9,21,34,47].

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Table 1
A formal context for describing the characteristics of the reduction techniques.

	Characteristics					
	CL is needed	Consistency is needed	For general dataset	Output all reducts	Information loss	Heuristic
Wei et al. [39] (a)	×	×		×		
Wei et al. [39] (b)	×	×		×	×	
Wang & Zhang [38]	×	×		×	×	
Wu et al. [43]		×		×		
Li et al. [13]	×	×			×	×
Li et al. [14]	×		×	×		

In order to implement a special decision making with FCA, Zhang and Qiu [49] introduced the notion of a formal decision context by dividing the attributes of a formal context into two parts: conditional attributes and decision attributes. Then, decisions in formal decision contexts were made by all kinds of rules, such as decision implications [28], granular rules [43], and decision rules [13,14,30]. In fact, formal decision contexts can be viewed as an extension of formal contexts. It is of practical importance to make such an extension. The reasons are summarized as follows: (1) it is often encountered in the real world that relational databases include decision attributes; (2) formal decision contexts allow us to give a more particular explanation of the induced implications since certain types of rules can be derived from a formal decision context to meet users' different requirements; (3) formal decision contexts can help us to deal with the basic issues in rough set theory [25] from the viewpoint of FCA, such as lower and upper approximation operators [33], reduction [6] and attribute dependency [11] in decision tables. The third reason is in accordance with the fact that formal decision contexts are beneficial to the comparison and combination of FCA with rough set theory [40].

As far as we know, knowledge reduction is one of the most important issues in the study of formal decision contexts, and its aim is to avoid the redundancy of attributes while preserving the predefined properties [36,48]. Up to now, many reduction methods (e.g. [13,14,26,29,38,39,43]) have been obtained. For example, Wei et al. [39] put forward two consistencies between the concept lattice of the formal context (G, M, I) and that of (G, N, J), where G is an object set, M is a conditional attribute set, and N is a decision attribute set of a formal decision context. More specifically, the first consistency means that for any concept of (G, N, J), there exists a concept of (G, M, I) such that their extents are the same. The second consistency means that for any concept of (G, N, J), there exists a concept of (G, M, I) such that the extent of the latter concept is included in the extent of the former one. Then, based on the defined consistencies, two reduction methods were developed for consistent formal decision contexts. More precisely, the first reduction method is to avoid the redundancy of attributes while preserving the non-redundant decision rules, but the second one only preserves the number of decision rules. Wang and Zhang [38] presented a different one whose aim is to avoid the redundancy of attributes while preserving the number of feedforward non-redundant decision rules. Moreover, Wu et al. [43] proposed a novel reduction method whose aim is to avoid the redundancy of attributes while preserving the granular rules. In addition, Li et al. [13] designed another reduction method whose aim is to avoid the redundancy of attributes while preserving the number of non-redundant decision rules. Note that the above five reduction methods are only suitable for consistent formal decision contexts. In other words, they will become invalid for inconsistent formal decision contexts. Motivated by this problem, Li et al. [14] gave a new reduction method which can be suitable for general formal decision contexts, and they used it to derive more compact decision rules from a formal decision context. In fact, the existing reduction methods can be classified, according to different types of concept lattices on which they are based, into three categories: the first category was performed by Wille's concept lattice (e.g. [13,14,38,39,43]), the second by the concept lattice induced by axialities (e.g. [26]), and the third by the object-oriented and property-oriented concept lattices (e.g. [29]). Since the second and third categories were obtained by extending the results of the first category into the generalized concept lattices, it is sufficient to discuss the reduction methods in the first category (i.e., the six reduction methods in [13,14,38,39,43] whose characteristics are summarized in Table 1, where CL is the abbreviation of "concept lattice", and a and b respectively represent the first and second reduction methods in [39]) when we study the existing reduction methods.

Although many reduction methods have been proposed for formal decision contexts, how to select a suitable one for a given specific case still needs to be studied. Up to now, little work has been devoted to this problem. In fact, to achieve this task, it is necessary to clarify the inherent relationship among the existing reduction methods. Our paper is going to investigate the relationship among the reduction methods in formal decision contexts from three aspects: providing a rule-based review of them, comparing their consistencies, and making a comparison of their reductions. The first aspect reveals the type of rules that each of them can preserve, and the other two can help users to select an appropriate reduction method to meet their requirements.

The rest of this paper is organized as follows. Section 2 recalls some basic notions related to formal concept analysis and introduces several types of rules in formal decision contexts. Section 3 provides a rule-based review of the reduction methods in formal decision contexts. Section 4 makes a comparison of the consistencies defined in the existing reduction methods. Section 5 investigates the relationship among the reductions under the five kinds of consistent formal decision contexts. The paper is then concluded with a brief summary and an outlook for further research.