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Contraction in propositional logic

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A R T I C L E I N F O

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The AGM model for the revision and contraction of belief sets provides rationality postulates for each of the two families of change operators. In the context of finite propositional logic, Katsuno and Mendelzon pointed out postulates for the revision of belief bases which correspond to the AGM postulates for the revision of beliefs sets. In this paper, we present postulates for the contraction of propositional belief bases which correspond to the AGM postulates for the contraction of belief sets. We highlight the existing connections with the revision of belief bases in the sense of Katsuno and Mendelzon thanks to Levi and Harper identities. We also present a representation theorem for contraction operators for propositional belief bases.

1. Introduction

Belief change has been studied for many years in philosophy, databases, and artificial intelligence. The AGM model, named after its three initiators Carlos Alchourrón, Peter Gärdenfors and David Makinson, is the main formal framework for modeling belief change [1]. Its key concepts and constructs have been the subject of significant developments [2–5]. Alchourrón, Gärdenfors and Makinson pointed out some postulates and representation theorems thereby establishing the basis for a framework suited to the belief change issue when beliefs are expressed using the language of any Tarskian logic, and given as belief sets (i.e., sets of beliefs closed under the consequence relation). Tarskian logics consider abstract consequence relations, that satisfy inclusion, monotony and idempotence (and the AGM framework adds also to them the supraclassicality, compactness and deduction conditions).

Katsuno and Mendelzon [6] presented a set of postulates for revision operators of belief bases in the framework of finite propositional logic. Generally speaking, a belief base is simply a non-deductively closed set of formulas. In [6] and in the present paper, a belief base can also be viewed as a single formula (the conjunction of its elements). Especially, belief base contraction could also be referred to as formula-based contraction. This departs from many works where the term "belief base contraction" is used for denoting syntax-dependent belief change [3].

Katsuno and Mendelzon [6] gave a representation theorem for revision operators in terms of faithful assignments.[1] This representation theorem is important because it is at the origin of the main approaches to iterated belief revision [4].

Revision and contraction operators of belief sets are closely related, as reflected by Levi and Harper identities. These identities can be used to define contraction operators from revision operators and vice versa. So works on contraction in

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[1] Such assignments correspond to a specific case of Grove's systems of spheres [7].

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the context of finite propositional logic might be expected. However, as far as we know, this issue has not been formally investigated up to now.

The aim of this paper is to define operators for propositional contraction of belief bases matching Katsuno and Mendelzon's revision operators and to check that these operators offer some expected properties. In the following, we give a set of postulates for contraction operators of belief bases in the framework of finite propositional logic, and establish a corresponding representation theorem.

The obtained results are not very surprising, since most of them can be obtained as corollaries of existing theorems (especially previous representation theorems and Levi and Harper identities). Nonetheless we consider that proving these results directly (and not as a byproduct of existing results) is important for a number of contexts where the whole AGM machinery cannot be applied directly.

Let us focus on two of them, which are quite significant. First, many works from the recent past years have been dedicated to the design and the study of belief revision and contraction operators in some weaker logical settings than the classical one, for instance Horn logic [8–11]. The obtained results form an important step towards the definition of change operators suited to Description Logics [12], or for other fragments of classical logic [13]. Indeed, in these weaker settings, there is no guarantee that the results from standard AGM theory still hold. In particular, some characterizations of belief change constructions that are equivalent (obtained by representation theorems) in the classical case are not equivalent any more. Furthermore, Harper and Levi identities do not necessarily hold (see [14] for the Horn case).

The second context we want to take as example is iterated change. Iterated revision has been extensively studied so far. This led to the development of revision operators on epistemic states, that are an epistemic representation that is more expressive than belief sets [4,15–17]. These works about iterated revision are based on Katsuno and Mendelzon's approach to the revision of propositional belief bases [6]. However, as far as we know, no study of what could be the corresponding iterated contraction operators has been conducted so far. This is not a straightforward task given that the Harper and Levi identities are not easily expressible on epistemic states. Accordingly, establishing direct formulations and proofs of results on propositional contraction appears as an important step for designing such iterated contraction operators (again one can not take advantage of the usual identities and representation results in this case).

The rest of the paper is organized as follows. In Section 2, some formal preliminaries are presented. In Section 3, the AGM and KM settings for belief contraction and revision are recalled. In Section 4, the standard connection between belief sets and belief bases is recalled as well. In Section 5, we define postulates that a contraction operator for propositional belief bases should satisfy. In Section 6 the correspondence between contraction of belief sets and contraction of belief bases is investigated; we check, using Levi and Harper identities, that there is a connection between propositional revision operators satisfying Katsuno and Mendelzon postulates and propositional contraction operators satisfying our postulates. Section 7 gives a representation theorem for the contraction of belief bases. Section 8 illustrates the representation theorem by defining a Dalal contraction operator. We conclude and discuss some perspectives for future work in Section 9.

2. Preliminaries

We consider a finite propositional language L built up from a (finite) set of propositional symbols P and the usual connectives. $\bot$ (resp. $\top$) is the Boolean constant `false` (resp. `true`). Formulas are interpreted in the standard way. $\vdash$ denotes logical deduction, and $\equiv$ denotes logical equivalence. $\mathcal{P}(L)$ denotes the power set of L, i.e., the set of all subsets of L.

A belief base is a finite set of propositional formulas $\{\varphi_1, \ldots, \varphi_n\}$. We suppose in this paper that a belief base is conjunctively interpreted, i.e., $\{\varphi_1, \ldots, \varphi_n\}$ is equivalent to $\varphi = \varphi_1 \wedge \ldots \wedge \varphi_n$ (this is a usual harmless assumption when one supposes irrelevance of syntax (cf. postulate (C5)).

A belief set is a deductively closed set of formulas. This set is infinite, thus not convenient for an effective representation. Given any element K of $\mathcal{P}(L)$, $Cn(K)$ denotes the deductive closure of K. When K is a singleton $\{\varphi\}$, or more generally, when K is equivalent to a formula $\varphi \in L$, we also write $Cn(\varphi) = \{\psi \in L \mid \varphi \vdash \psi\}$ to denote the set of classical consequences of φ.

Fortunately, one can associate with any belief base φ a belief set K that is the set of all its consequences $K = Cn(\varphi)$. Reciprocally, we can always represent a belief set by a (finite) belief base consisting of a single formula (it is enough to select one representative of the logically strongest formulas in the belief set to get such a belief base).

An interpretation I is a mapping associating every symbol from P with a truth value. If φ is a formula from L, then $Mod(\varphi)$ denotes the set of its models. Conversely if M is a set of interpretations over P, then α_M denotes the formula (unique, up to logical equivalence) the models of which are those of M.

Given a preorder (i.e., a reflexive and transitive relation) $\leq_\varphi$ over the set of interpretations, $<_\varphi$ is its strict part defined by $I <_\varphi J$ if and only if $I \leq_\varphi J$ and $J \not\leq_\varphi I$ and $\simeq_\varphi$ is the associated equivalence relation defined by $I \simeq_\varphi J$ if and only if $I \leq_\varphi J$ and $J \leq_\varphi I$. $\min(X, \leq_\varphi)$ denotes the set of minimal elements of X for $\leq_\varphi$, i.e., $\min(X, \leq_\varphi) = \{x \in X \mid \nexists y \in X$ such that $y <_\varphi x\}$.