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Pricing and hedging in a single period market with random interval valued assets

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ABSTRACT

In this article, a new financial market model, in which securities have random interval valued payoffs, is proposed. As an extension of traditional random market model, some concepts, such as robust arbitrage opportunities, risk-neutral pricing measures and robust replicative strategies, are given and discussed parallel to those in traditional market analysis. With these new concepts, problems of pricing and hedging are analyzed. It is shown that the requirement of no robust arbitrage opportunities is equivalent to the existence of risk-neutral pricing measures. Taking no robust arbitrage as the valuation principle, the problem of pricing a contingent claim with random interval valued payoff is discussed. All no robust arbitrage prices of the claim form an interval, whose endpoints can be got from the risk-neutral pricing measures or from robust replicative strategies.

1. Introduction

In traditional financial analysis, a market is modeled by a probability space $(\Omega, \mathcal{F}, \mathrm{P})$, with the state set Ω, the information structure $\mathcal{F}$, and a given probability measure P. All securities are represented by their random values/payoffs, which are random variables in the given space. Some problems, such as pricing, utility maximization, portfolio selection, hedging and risk measuring are well treated. In a market affected by vagueness and ambiguity [1], a Knightian uncertain market model is used, where P is replaced by a capacity, or a set of probability measures. The classical expected utility model has been extended to robust expected utility to fit different priors. Some problems, such as portfolio selection, utility maximization and market equilibrium, will be affected because different meanings will be obtained under different probability measures. However, the problem of pricing is unaffected. Under no-arbitrage principle, the well-known pricing result can be stated as in [2,3]: "the price of a security is the discounted expectation of its future value under equivalent martingale measures". We can see that, in an uncertainty model, if all measures in the measure set are equivalent, the set of equivalent martingale measures will not change whatever real measure is chosen. We only need to pick up one measure to determine equivalent martingale measures.

Most popular tools to handle quantification of uncertainty belong to the region of imprecise probability. Many different but related models have been put forward, as discussed and compared in [4]. Among these models, coherent upper and lower previsions, based on comprehensive foundations put forward by Walley [5], provide many rationality criteria for reasoning with subjective probabilities, as surveyed in [6]. In a classical incomplete financial market, there exist more risk-neutral pricing measures. Endpoints of the no-arbitrage price interval of a contingent claim, that can't be replicated by fundamental securities, can be represented as the maximal/minimal expectation of its discounted payoff under the set of pricing measures. So two endpoints of the price interval are special coherent upper and lower previsions. In the special issue in International Journal of Approximate Reasoning, with the first introductory article [7], we can see that many

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models with imprecise probability methods have been used in financial analysis under different uncertainties. Among these articles, [8] has compared concepts of coherent previsions and arbitrage-free asset pricing. It was shown that these two concepts were similar but not identical. [9] proposed an American option pricing model with imprecise volatility. In that multi-period binomial tree model, the uncertainty on the volatility was modeled by means of a possibility distribution. [10] defined coherent lower or upper previsions in terms of admissible gains to investigate the well-known betting scheme. The correspondence of the scheme for imprecise previsions with real-world situations was analyzed. Toward the problem of option pricing in an incomplete market based on partial information, [11] described a new valuation approach by selecting a risk-neutral probability minimizing the pseudo-distance between the partial conditional probability assessment and the convex set of all possible risk-neutral probabilities.

There are also some cases in which the market will be affected by imprecise observations [12,13], information insufficiency [14], and experts' subjective opinions [15]. More uncertain tools or techniques, such as interval numbers, fuzzy numbers and fuzzy random variables, have been used for market modeling. [16] and [17] dealt portfolio selection problem, where returns were interval numbers. [18–20] analyzed the problem of pricing European and American options in uncertain environments, where some key factors were fuzzy or fuzzy random valued.

When uncertainties are caused by imprecise observation of random variables, random sets are applicable uncertain tools, just as explained in Kruse and Meyer [13]. A random set is a generalized random variable which is set-valued at every possible state. We can see successful applications of random set techniques in different fields, such as in economy [21], and in geometry [22]. Applications of random sets in mathematical finance and econometrics are surveyed in Chapter 17 of [23], where we can see wide-spread applications to trading with transaction costs, risk measures, option prices, and partially identified econometric models. For example, in real markets, the asset price will be random interval valued because transaction costs make each asset traded at two different prices: bid price offered by the buyer and ask price by the seller. In many situations, we can't identify unknown parameters in probability distributions perfectly because of incomplete financial data. In such case, the best way is to identify a certain set, to which parameters belong.

This article will extend the traditional financial market model to a model in which securities have random interval payoffs, but with the underlying probability space, especially the probability measure, fixed. The uncertainties discussed here are not from probability measures, but from realizations of random variables. A random interval is a special random set-valued as a closed interval at any possible state. Given a probability space, a random interval can be roughly defined as an interval whose two endpoints are random variables. As explained in [24], due to some imprecision in the observation or to existence of missing data, the images of a random variable ξ at state $\omega \in \Omega$ can't be known precisely. The suitable approach in this situation is to give upper and lower bounds of its images. If we denote the two bounds as $\xi^L(\omega)$ and $\xi^U(\omega)$, we get a random interval $[\xi^L(\omega), \xi^U(\omega)]$. In this article, random intervals are used to measure securities, which will lead to new concepts and arguments, compared to traditional financial analysis.

A binomial tree model has been given in [25], where the stock price at each state takes imprecise values. This model will be covered by this article if such imprecise values are interval numbers. In this reference, the vagueness in the stock price movements was modeled by fuzzy numbers. Based on the risk-neutral valuation approach, weighted interval valued probabilities were derived as pricing measures. Such measure was still in the range of imprecise probability. Also in [26], random interval and fuzzy random market models with finite states are proposed, so that the interval or fuzzy valued payoff matrix was applied to model securities. Under the acceptability argument, the valuation rules were discussed. Different to both references, a general market model with random interval valued payoffs will be proposed here. Also we will give a new concept of robust arbitrage for further discussion. A normal probability measure, not an uncertain probability as in [25], will be derived as a pricing measure. Also we will find an acceptable market in [26] is a stronger requirement than a market with no robust arbitrages, which will be used as a valuation principle in this article.

This article is arranged as follows. Some preliminaries on interval numbers and random intervals applicable to our discussion are covered in Section 2. In Section 3, a financial model with securities of random interval valued payoffs is given. Two concepts of robust arbitrage opportunity and risk-neutral pricing measure are proposed. The relations between these two concepts are discussed. Under no robust arbitrage argument, a price interval of a contingent claim with random interval valued payoff is got in Section 4. Concepts of robust replicative strategies are proposed in Section 4, too. In Section 5, an illustrative example is given. Conclusions and further topics are brought in Section 6.

2. Preliminaries on random intervals

Here are some references on fundamentals of random sets and random intervals, such as in [24,27,28]. Formally, a random set is a multi-valued mapping satisfying some measurability condition, as discussed in [24]. Because only random intervals are used as a tool for market modeling in this article, we only list some basic definitions and operations, from materials in [24] and [29].

Definition 1. An interval number is a closed interval in $\mathbb{R}$: $\bar{a} = [a^L, a^U]$.

An interval number is just a set of numbers. When we can't tell the value of a quantity for sure, we can get the bounds of the quantity, and describe the quantity as an interval number. Throughout this article, the superscripts L and U will be used to express the lower and upper endpoints of an interval number, respectively.