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A fuzzy real-time temporal logic

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ABSTRACT

High-level descriptions of real-time systems often use fuzzy notions of time that are left open to domain specific interpretations. In order to verify that a given implementation conforms to such loosely defined specifications, the typical approach is to verify the implementation to be correct within well defined limits of time tolerance. This approach determines whether the real-time requirements are met, but does not reflect *how well it is met*.

Our goal in this paper is to prescribe the development of timed specifications using fuzzy notions of time, and to present a methodology for computing the *quality of satisfaction* of the specification on a given implementation using domain specific fuzzy membership functions. With this objective, we combine the notions of real-time interval temporal logic (like Metric Interval Temporal Logic) and fuzzy logic to derive FRTL, a *fuzzy real-time temporal logic*. The novelty of the proposed logic is in introducing the notion of fuzzy time intervals into the core fabric of conventional metric temporal logic. We present a method for evaluating the fuzzy truth of FRTL properties on finite traces. We discuss the motivation of computing the fuzzy truth towards evaluating the *quality of control* in time critical embedded control system applications. We also show that two important related problems from the domain of mixed-signal design verification, are subsumed by the proposed framework of analysis.

1. Introduction

Real-time and hybrid systems often have timing requirements, whose interpretation varies on a case-to-case basis, and are therefore not expressible with a logic formula using precise time intervals. Real-time logics, like Metric Interval Temporal Logic (MITL) [2], are capable of expressing temporal requirements that involve well defined time intervals. Parametric temporal logic (PLTL) [1] allows the specification of properties using parameterized time constraints. However none of these logics allow the specification of properties for which the timing requirement is fuzzy.

There is a significant difference between expressing timing requirements through *well defined time intervals* and *fuzzy time intervals*. A property using well defined time intervals has a Boolean truth – it is true if the timed functionality is met within the specified interval and false otherwise. A property using fuzzy time intervals will have a fuzzy truth (which is a real number), depending on *how well* the timed functionality was met within the fuzzy interval. The latter allows us to reason about the *quality of satisfaction*, with promising ramifications towards defining the *quality of real-time control* in embedded systems.

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In this paper, we combine the notions of real-time interval temporal logic and fuzzy logic to derive a fuzzy real-time temporal logic called FRTL. As opposed to existing literature, FRTL introduces the notion of fuzzy intervals into the core fabric of dense real-time temporal logic. We present the formal syntax and semantics of FRTL, and present an algorithm for computing the truth of FRTL properties on finite real-time traces.

We argue that the proposed approach is promising towards evaluating the quality of control for time critical embedded systems. We also study two related problems from the domain of mixed-signal design verification, namely robustness and coverage, and show that these problems are subsumed by the proposed framework.

The paper is organized as follows. Section 3 studies the background of real-time logic, and presents an extension of MITL, called AMS-LTL [17], which we further extend to derive FRTL in subsequent sections. Section 4 presents formal syntax and semantics of FRTL. Section 5 outlines a method for computing fuzzy truth of FRTL assertions from finite execution trace. Section 6 demonstrates expressibility of FRTL with a test case from the domain of automotive systems. Section 7 presents two important problems from the domain of circuit verification to show their containment into FRTL.

2. Related work

A fuzzy logic enables one to express non-linear behaviors in terms of simple linguistic expressions which are interpreted with respect to domain specific membership functions. Previous studies on fuzzy temporal logic [12] combined the power of fuzzy logic and discrete-time temporal logic [20] to provide frameworks for expressing fuzzy temporal behaviors in simple and linguistic ways.

The authors of [9] elaborate the vagueness/uncertainty while representing time boundaries and intervals. There the authors proposed a fuzzy way to express the vagueness while representing a time instance. We use a similar notion in the proposed interval temporal logic. This allows us to combine temporal operators with uncertain interval annotations. The authors of [8] propose an extension of Allen's relational calculus based on fuzzy comparators expressing linguistic tolerances. They introduce fuzzy relations like *before*, *after*, *meets* among the time intervals in order to express linguistic temporal relationship between time intervals, and propose an approach of temporal reasoning based on fuzzy Allen relations. In an earlier paper [10], the author proposed a similar notion of fuzzy intervals for representing event based fuzzy knowledge. Fuzzification of Allen's temporal interval relation has also been discussed in [22]. The author of [18] also proposes to express fuzzy intervals in terms of polygons over the integer coordinate, and develops a library to represent and manipulate fuzzy intervals.

Fuzzy notion of temporal logic was introduced by the authors of [23], extending classical propositional calculus of Lukasiewicz with classical temporal operators (like *next*, *until*). The authors of [24] propose a fuzzy temporal logic scheme for representation and control of fuzzy dynamic systems. Other works on fuzzy temporal logic include [6,25,5].

The approach proposed in this paper has several notable differences with the existing body of literature. Firstly, we introduce the notion of fuzzy time intervals into the fabric of dense real-time temporal logic as opposed to discrete-time temporal logic. The density of time has an important role in determining the quality of satisfaction of a fuzzy real-time property. Secondly, the underlying temporal logic facilitates specification development over the use of algebraic specifications, or those based on relational calculi. The fuzzy intervals, as a part of the logic formulas, facilitates the verification engineer to write approximate timing behaviors of real-time systems. Fuzzy truth of an approximate specification gives a goodness measure for the system.

3. Foundations

Propositional temporal logic extends Boolean logic by allowing us to relate the truth of Boolean propositions in different time worlds. The syntax of Linear Temporal Logic (LTL) [20] is defined over a set of *atomic propositions*, $\mathcal{AP}$, as follows:

1. Each $p \in \mathcal{AP}$ is a LTL formula.
2. If f and g are LTL formulas, then so are $\neg f$, $f \wedge g$, $\mathcal{X} f$ and $f \, \mathcal{U} \, g$.

$\mathcal{X}$ represents the next-time operator and $\mathcal{U}$ represents the *until* operator. The formula $\mathcal{X} f$ is *true* in a time world iff f is *true* in the next-time world. The formula $f \, \mathcal{U} \, g$ is *true* in a time world iff g is *true* in some future time world and f is *true* in all time worlds in between. LTL forms the backbone for assertion language standards like SystemVerilog Assertions (SVA) [13] and Property Specification Language (PSL) [14] adopted by the industry.

Though propositional temporal logic like LTL can express sequences of *events*, it cannot express real-time constraints between *events*. Real-time temporal logics are extensions of propositional temporal logic which allow real-time constraints. These logics are broadly divided into those that express constraints over discrete time and those that express constraints over continuous time. The former is what is needed in digital design verification, where real-time is expressed in multiples of clock cycles, while the latter is what we need for expressing mixed-signal properties.

There is a significant volume of existing literature on real-time logic for continuous domain properties and on the algorithms for checking such properties. Such logics include Metric Temporal Logic (MTL) [3], Metric Interval Temporal Logic (MITL) [2], TPTL [4], and LTLC [15]. Some of them, such as MITL, have recursive satisfiability problem [3], whereas many are undecidable when time is dense.

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