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Fuzzy analytic hierarchy process: A logarithmic fuzzy preference programming methodology ☆

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ABSTRACT

Fuzzy analytic hierarchy process (AHP) proves to be a very useful methodology for multiple criteria decision-making in fuzzy environments, which has found substantial applications in recent years. The vast majority of the applications use a crisp point estimate method such as the extent analysis or the fuzzy preference programming (FPP) based nonlinear method for fuzzy AHP priority derivation. The extent analysis has been revealed to be invalid and the weights derived by this method do not represent the relative importance of decision criteria or alternatives. The FPP-based nonlinear priority method also turns out to be subject to significant drawbacks, one of which is that it may produce multiple, even conflict priority vectors for a fuzzy pairwise comparison matrix, leading to entirely different conclusions. To address these drawbacks and provide a valid yet practical priority method for fuzzy AHP, this paper proposes a logarithmic fuzzy preference programming (LFPP) based methodology for fuzzy AHP priority derivation, which formulates the priorities of a fuzzy pairwise comparison matrix as a logarithmic nonlinear programming and derives crisp priorities from fuzzy pairwise comparison matrices. Numerical examples are tested to show the advantages of the proposed methodology and its potential applications in fuzzy AHP decision-making.

1. Introduction

As a practical yet popular methodology for dealing with fuzziness and uncertainty in multiple criteria decision-making (MCDM), fuzzy analytic hierarchy process (AHP) has found huge applications in recent years. Since fuzzy judgments are easier to provide than crisp judgments, it can be concluded that fuzzy AHP will find more applications in the near future. The use of fuzzy AHP for multiple criteria decision-making requires scientific approaches for deriving the weights from fuzzy pairwise comparison matrices. Existing approaches for fuzzy AHP weight derivation can be classified into two categories, one of which is to derive a set of fuzzy weights from a fuzzy pairwise comparison matrix, while the other is to derive a set of crisp weights from a fuzzy pairwise comparison matrix. The approaches for deriving fuzzy weights from fuzzy pairwise comparison matrices mainly include the geometric mean method [7], fuzzy logarithmic least-squares methods (LLSM) [2,62,66], Lambda–Max methods [19,63] and the linear goal programming (LGP) method [64]. The approaches for deriving crisp weights from fuzzy pairwise comparison matrices include the extent analysis [18] and the fuzzy preference programming (FPP) based nonlinear method [45].

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Since fuzzy weights are not as easy to compute as crisp ones, our literature survey shows that the vast majority of the fuzzy AHP applications uses a simple extent analysis method proposed by Chang [18] for fuzzy AHP weight derivation for simplicity. However, such an extent analysis method has been revealed by Wang et al. [68] to be invalid and the weights derived by this method do not represent the relative importance of decision criteria or alternatives at all. It has led to a significantly large number of misapplications in the literature [1,3–6,8–17,19–44,47–49,50–61,67,69,70]. Apparently, its usage as a weight derivation method should be rejected. The FPP-based nonlinear priority method proposed by Mikhailov [45] has also found some applications in recent years [21,65]. Unfortunately, such a method also turns out to be subject to some significant drawbacks. For example, it may produce multiple, even conflict priority vectors for a fuzzy pairwise comparison matrix, leading to distinct conclusions. This non-uniqueness in solutions damages its applications as a priority method for fuzzy AHP.

To provide a valid yet practical priority method for fuzzy AHP, this paper proposes a logarithmic fuzzy preference programming (LFPP) based methodology for fuzzy AHP priority derivation, which formulates the priorities of a fuzzy pairwise comparison matrix as a logarithmic nonlinear programming and derives crisp priorities from fuzzy pairwise comparison matrices. The paper is organized as follows. Section 2 briefly reviews the FPP-based nonlinear priority method and illustrates its non-uniqueness in solutions. Section 3 proposes the LFPP-based methodology for fuzzy AHP weight derivation. Its validities are tested with numerical examples in Section 4. The paper concludes in Section 5.

2. The FPP-based nonlinear priority method and its non-uniqueness in solutions

Suppose the decision maker (DM) provides fuzzy judgments instead of precise judgments for a pairwise comparison matrix. For example, it could be judged that criterion i is between l_{ij} and u_{ij} times as important as criterion j with m_{ij} being the most likely times. Then, a fuzzy pairwise comparison matrix can be expressed as

$$\widetilde{A} = (\tilde{a}_{ij})_{n\times n} = \begin{bmatrix} 1 & (l_{12}, m_{12}, u_{12}) & \cdots & (l_{1n}, m_{1n}, u_{1n}) \\ (l_{21}, m_{21}, u_{21}) & 1 & \cdots & (l_{2n}, m_{2n}, u_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (l_{n1}, m_{n1}, u_{n1}) & (l_{n2}, m_{n2}, u_{n2}) & \cdots & 1 \end{bmatrix}, \tag{1}$$

where $l_{ij} = 1/u_{ji}$, $m_{ij} = 1/m_{ji}$, $u_{ij} = 1/l_{ji}$ and $0 < l_{ij} \leqslant m_{ij} \leqslant u_{ij}$ for all $i,j = 1,\ldots,n$; $j \neq i$. To find a crisp priority vector $W = (w_1,\ldots,w_n)^T > 0$ with $\sum_{i=1}^{n} w_i = 1$ for the fuzzy pairwise comparison matrix in (1), Mikhailov [45] introduces the following membership function for each fuzzy judgment in $\tilde{A}$:

$$\mu_{ij}\left(\frac{w_i}{w_j}\right) = \begin{cases} \frac{(w_i/w_j)-l_{ij}}{m_{ij}-l_{ij}}, & \frac{w_i}{w_j} \leqslant m_{ij}, \\ \frac{u_{ij}-(w_i/w_j)}{u_{ij}-m_{ij}}, & \frac{w_i}{w_j} \geqslant m_{ij}, \end{cases} \tag{2}$$

where $\mu_{ij}(w_i/w_j)$ is the membership degree of w_i/w_j belonging to the fuzzy judgment $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$. Let

$$\lambda = \min\left\{\mu_{ij}(w_i/w_j) \mid i = 1,\ldots,n-1;\ j = i+1,\ldots,n\right\}. \tag{3}$$

Then, λ is the minimum membership degree to which the crisp priority vector satisfies each fuzzy pairwise comparison. It is hoped that the priority vector should be able to maximize the DM's satisfaction. For this hope, Mikhailov [45] established the following FPP-based nonlinear priority model, which is an extension of the FPP priority method for crisp pairwise comparison matrix [46] in fuzzy environments:

$$\begin{aligned} &\text{Maximize} \quad \lambda \\ &\text{Subject to} \begin{cases} \mu_{ij}(w_i/w_j) \geqslant \lambda,\ i = 1,\ldots,n-1;\ j = i+1,\ldots,n, \\ \sum_{i=1}^{n} w_i = 1, \\ w_i \geqslant 0,\ i = 1,\ldots,n, \end{cases} \end{aligned} \tag{4}$$

which can be equivalently expressed as

$$\begin{aligned} &\text{Maximize} \quad \lambda \\ &\text{Subject to} \begin{cases} -w_i + l_{ij}w_j + \lambda(m_{ij} - l_{ij})w_j \leqslant 0,\ i = 1,\ldots,n-1;\ j = i+1,\ldots,n, \\ w_i - u_{ij}w_j + \lambda(u_{ij} - m_{ij})w_j \leqslant 0,\ i = 1,\ldots,n-1;\ j = i+1,\ldots,n, \\ \sum_{i=1}^{n} w_i = 1, \\ w_i \geqslant 0,\ i = 1,\ldots,n. \end{cases} \end{aligned} \tag{5}$$

If the optimal objective value $\lambda^* > 0$, then the optimal solution $w_1^*,\ldots,w_n^*$ satisfy $l_{ij} \leqslant w_i/w_j \leqslant u_{ij}$; otherwise, there exists strong inconsistency among the fuzzy judgments and the optimal solutions only approximately satisfy the fuzzy pairwise comparison matrix.