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Moment-SOS relaxation of the medium term hydrothermal dispatch problem

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ABSTRACT

This paper analyzes the application of moment-SOS (sum of squares) relaxation to solve the medium term hydrothermal dispatch (MTHD) problem for predominantly hydro systems. The optimal dispatch is formulated as a two stage stochastic programming problem, which is nonconvex due to constraints that represent hydro power production, and may have multiple solutions. The size and conditioning of the problem pose challenges to the use of the semidefinite programming (SDP) techniques that are discussed in the text. The size of the problem restricts this study to first order moment relaxations. Therefore, to improve the quality of relaxed solutions, additional meaningful constraints are introduced in the SDP problems. Subsequently, the problem re-scalded to achieve numerical stability in SDP relaxation. Results obtained for different test systems, most of them being partitions of the Brazilian hydroelectric system, indicate that the procedures used to improve the quality of SDP solutions are effective.

1. Introduction

Operational planning studies of hydrothermal systems are carried out considering a long term horizon to ensure the most efficient usage of water resources. These studies take into consideration dispatch procedures. In countries where system operators are responsible for the centralized dispatch, generation targets are determined for all the plants whose capacities are above a specified limit, so that expected operating costs are minimized considering hydro generation availability over the planning period. Due to the complexity of such task, mathematical models used in long term studies usually do not represent the power plants or the transmission system in detail. Thus, medium and short term analysis must be executed [1]. This paper is concerned with the task of planning the operation of a predominantly hydro system considering a medium term horizon. Uncertainty pertaining to water inflows is considered and the dispatch is modelled as a two stage stochastic programming problem. The paper analyses the application of semidefinite programming (SDP), specifically, moment-SOS relaxation [2,3], to solve the problem.

SDP has been applied to a wide variety of power system studies, notably to the optimal power flow problem [4–12], but also to state estimation [13,14], to calculate the optimal unit commitment of thermal plants [15,16], to suppression of inter-area oscillation [17], optimal line switching [18] and in operational planning of hydrothermal systems [19–21]. The last three references study different formulations of the optimal dispatch problem considering planning horizons of one day up to one week. The optimal dispatch is represented by deterministic models. In [19] on/off status of thermal units is taken into consideration and the problem is formulated with continuous and binary variables and linear constraints. On the other hand, in [20], online thermal plants are considered known and quadratic functions represent input-output characteristics of hydro units, therefore, a quadratically constrained quadratic problem is solved. In these two studies, SDP is directly applied to solve the problem. However, in [21], SDP is adopted to set lower bounds to solution estimates obtained in the iterations of a branch and bound algorithm, which calculates the optimal commitment of hydro units considering AC transmission constraints.

The medium term hydrothermal dispatch (MTHD) problem is usually formulated for a planning horizon of one or two years. This problem has been extensively studied and has stochastic [22–26] and deterministic [27–31] formulations, depending on whether the random nature of natural water inflows is taken into consideration. The nonlinear relation between hydro power generation and the water heads and/or water releases of the plants is not considered in most stochastic models, but is represented in the deterministic models by different polynomials. Dynamic programming and decomposition techniques have been used to solve the stochastic problem. On the other hand, the deterministic problem has been solved via Lagrangian relaxation, network flow programming [27–29] and interior point methods [30,31]. The methods used to solve the nonlinear MTHD problem have local convergence characteristics. The present paper formulates the MTHD problem for a predominantly hydro system, takes into consideration the random nature of water inflows and expresses hydro power generation

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Notation

Indices

i, m	Hydro plant indices
s	Scenario index
t	Time period index

Functions

f	Objective function of the dispatch problem
g_j	*j-th* Constraint of the dispatch problem
$g_{v_{i,t,s}}$	Volume of water of reservoir i as a function of water inflow, discharge and spillage rates at time period t and scenario s
$g_{\bar{v}_{i,t,s}}$	Average volume of water of reservoir i as a function of water inflow, discharge and spillage rates
$g_{l_{t,s}}$	Total generation of the system at time period t and scenario s
$g_{v_{i,T,s}}$	Volume of water stored in reservoir i at the end of the planning horizon as a function of water inflow, discharge and spillage rates
$(g_j)\mathbf{y}$	Constraint g_j expressed in terms of the vector of lifting variables
h_v, h_q	Forebay and afterbay elevations of the hydro plant
$L_y\{f\}$	Representation of f in the lifted space

Constants

α_{0_i}, α_{1_i}	Constant and linear coefficients of h_v for plant i
β_{0_i}, β_{1_i}	Constant and linear coefficients of h_q for plant i
c_0, c_1, c_2	Constant, linear and quadratic coefficients of the cost function
h	Duration of period t in hours
H	Number of hydro plants
k_i	Producibility constant of plant i
k_{1_i}, k_{2_i}, k_{3_i}	Coefficients of the power production function of hydro plant i
n	Number of variables of the hydrothermal dispatch problem
N	Order of the moment relaxation
N_s	Number of streamflow scenarios
p_{d_t}	System load at time period t
$\pi(s)$	Probability of occurrence of scenario s
$r_{i,t,s}$	Natural water inflow of reservoir i at time period t and scenario s
T	Number of periods in the planning horizon
v_i^{sp}	Target value for the amount of water stored in reservoir i at the end of the planning period
ω_j	Degree of constraint g_j

Variables and Variable Matrices

$\mathbf{M}_N(\mathbf{y})$	Moment matrix of order N
$p_{h_{i,t,s}}$	Power produced by plant i, at time period t and scenario s
$pt_{t,s}$	Thermal generation at period t and scenario s
$q_{i,t,s}$	Amount of water through the turbines of plant i, at period t and scenario s
$u_{i,t,s}$	Amount of spilled water in reservoir i, at period t and scenario s
$v_{i,t,s}$	Volume of water in reservoir i at the end of period t and scenario s
$\bar{v}_{i,t,s}$	Average volume of water of reservoir i, at period t and scenario s
$\mathbf{x}$	Vector of variables of the dispatch problem
$\mathbf{x}_s$	Vector of variables of the dispatch problem associated with scenario s
$\hat{\mathbf{x}}$	Vector of variables equal to $[1,\ x_1, \ldots, x_n,\ x_1^2,\ x_1x_2, \ldots, x_n^2, \ldots, x_n^N]^\top$
$\check{\mathbf{x}}_j$	Vector associated with the maximal clique
$\tilde{\mathbf{x}}_{i,t,s}$	Vector of variables equal to $[1,\ q_{i,t,s},\ u_{i,t,s},\ \bar{v}_{i,t,s},\ pt_{t,s}]^\top$
$\hat{\mathbf{X}}$, $\check{\mathbf{X}}$, $\tilde{\mathbf{X}}_{i,t,s}$	Matrices equal to $\hat{\mathbf{x}}\hat{\mathbf{x}}^\top$, $\check{\mathbf{x}}\check{\mathbf{x}}^\top$ and $\tilde{\mathbf{x}}_{i,t,s}\tilde{\mathbf{x}}_{i,t,s}^\top$, respectively
$\mathbf{y}$	Vector of lifting variables

Other Symbols

	j-th maximal clique of the correlative sparsity pattern graph
max	Superscript – indicates maximum
min	Superscript – indicates minimum
x^α	Monomial $x_1^{\alpha_1}x_2^{\alpha_2}\ldots x_n^{\alpha_n}$, with $\alpha = \alpha_1 + \alpha_2 + \ldots + \alpha_n$
$\top$	Superscript – indicates transpose
Ω_i	Set of reservoirs upstream of reservoir i

by second order polynomials. The dispatch is obtained by solving a quadratically constrained stochastic quadratic problem, which is nonconvex and my have multiple solutions. SDP relaxation is used to obtain global optimal solutions to this problem.

SDP is a powerful tool to solve optimization problems with global and local minima. However, the application of SDP to real life problems is challenging. The relaxation usually leads to very large lifted problems that require considerable computational effort to be solved. In addition, SDP usually provides lower bounds for the optimal solutions, which must be improved towards optimal solutions via systematic procedures. Furthermore, the effectiveness of SDP depends on the characteristics of the feasible set of the original problem. In particular, if the magnitudes of some components of an optimal solution for the problem are considerably larger (smaller) than one, the magnitude of some lifting variables can be huge (tiny), which can cause numerical difficulties [32]. It has also been pointed out that the recovery of feasible solutions from the relaxed ones is not simple if the problem has multiple solutions with the same globally optimal objective value [33]. A number of procedures have been developed that tackle these issues: systematic increase of the size of the lifted problem in order to improve the quality of the relaxed solution, thus defining a hierarchy of relaxations [3]; sparsity techniques and second order conic relaxation to reduce the number of variables of SDP problems [32,9,10,12]; inclusion of additional meaningful constraints to improve the quality of solution [34,35,16,18]; the use of penalty factors [11] and introduction of cuts to reduce the feasible region of the lifted problem [36]. When solving the medium term hydro power dispatch (MTHD) problem, tests have shown that the most important challenges to overcome are: the size of the lifted problem as the number of power plants and/or streamflow scenarios increase, the existence of many solutions with the same globally optimal objective value, and scaling issues.

This paper analyzes the effectiveness of some of the procedures designed to improve SDP performance when applied to the MTHD problem. First of all, additional meaningful constraints are introduced in the problem. This procedure was first employed by Sheralli to solve a quadratically constrained quadratic problems using a branch-and-bound algorithm in which the lower bounds are calculated via a reformulation-linearization technique (RLT) [34]. RLT combines linear constraints of the problem to generate new valid quadratic constraints, and the quadratic problem with the additional constraints is linearized by defining new variables. Lasserre analyzes some properties of RLT relaxation and shows that it is outperformed by moment relaxation for small problems [37]. However, in a more recent paper, Anstreicher shows that the addition of RLT constraints to SDP problems can produce relaxed solutions that are better than those provided by either technique used alone [38].