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# A new governor power flow formulation based on the current injections method

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ABSTRACT

This paper presents a new governor power flow formulation using the current injections equations expressed in rectangular coordinates (CIM). An augmented current injection formulation is proposed to incorporate the static response of the generation governor equations which are expressed as a function of the system frequency and the active power generations. As the frequency becomes a new system variable, a new equation to set the network reference angle to a given value is included, being this another important contribution of this paper. Then, this system of equations is linearized and solved using the Newton-Raphson method. The proposed methodology allows determining the frequency deviation due to a load variation as well as the distribution of the active losses among the generators. Additionally, a new concept of reference bus of the system is introduced, allowing any bus (load or generation bus) to be the slack bus of the system. A small test system of 6 bus was used to show the new slack bus concept and the New England 39 bus test system was used to show the effectiveness of the proposed methodology which was validated using the ANATEM (Electromagnetic Transients Program) from CEPEL.

## 1. Introduction

The power flow formulation is a steady state power system analysis tool widely used and cited in the literature [1–5]. The basic methodology aims to determine the voltage magnitudes and angles at all buses, given the network parameters, topology and the generation and load patterns. This tool is also used for more complex studies such as power system reliability, optimization, electric operational and expansion planning, stability analysis, among others [6–9].

The modeling of the control actions, in steady state, from the existing control devices in a power system is an important requirement in order to obtain simulation results as close as possible from the power system steady state reality [3,8–12].

Reference [10] presents a fast decoupled load flow in which the load frequency control of generators are modeled. Unfortunately, the control actions are represented in a alternate way between iterations. In other words it is not a full Newton solution and for this reason it may not converge specially for heavy load conditions [13].

In [14–18] the load frequency control was represented in the full Newton power flow formulation in polar coordinates. Unfortunately in the above formulations the sparse structure of the network is not preserved in the Jacobian matrix and the reference bus must be a generation bus because the corresponding reference angle increment variable is replaced by the frequency deviation.

The slack bus is traditionally represented by a large generation bus in which both the active and reactive powers are calculated at the end of the iteration process, by assuming that the corresponding voltage magnitude and the reference angle are previously defined. In other words, the swing bus power equations are excluded from the nonlinear system of equations to be solved. As a result, this bus becomes the angular system reference and is responsible to match the power unbalances throughout the system, including the power transmission losses. However, this formulation, which is very simple, does not represent the real operation of the power system, requiring additional tools based on active power participation factors [10,19], to distribute the power losses among the generations.

Reference [11] describes the current injections power flow formulation in which the current injections power flow equations are written as a function of voltages in rectangular coordinates. Then the system of equations are linearized and solved using the Newton-Raphson technique.

This paper presents a sparse augmented governor power flow technique in which the primary frequency regulation equations which are functions of the corresponding active power generations and system frequency as well as the angular system reference equation are incorporated in the current injections method [11], producing an

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augmented system of equations. The augmented system of equations is linearized and solved using the Newton-Raphson method.

## 2. Proposed methodology

### 2.1. The governor power flow formulation

The basic current injection power flow equations [11] are given by:

$$\frac{(P_{g_k}-P_{l_k})V_{r_k} + (Q_{g_k}-Q_{l_k})V_{m_k}}{V_{r_k}^2 + V_{m_k}^2} - \sum_{i\in\varphi_k}(G_{ki}V_{r_i}-B_{ki}V_{m_i}) = 0 \tag{1}$$

$$\frac{(P_{g_k}-P_{l_k})V_{m_k}-(Q_{g_k}-Q_{l_k})V_{r_k}}{V_{r_k}^2 + V_{m_k}^2} - \sum_{i\in\varphi_k}(G_{ki}V_{m_i} + B_{ki}V_{r_i}) = 0 \tag{2}$$

where:

| | |
|---|---|
| $\varphi_k$ | Set of buses directly connected to bus $k$; |
| $G_{ki}$, $B_{ki}$ | Real and imaginary parts of elements of nodal admittance matrix; |
| $P_{g_k}$ | Active power generation output at bus $k$; |
| $Q_{g_k}$ | Reactive power generation output at bus $k$; |
| $P_{l_k}$ | Active power load at bus $k$; |
| $Q_{l_k}$ | Reactive power load at bus $k$; |
| $V_{r_k}$, $V_{m_k}$ | Real and imaginary parts of voltage at bus $k$. |

The values of loads at a bus $k$, known as PQ bus, $k$, $P_{l_k}$ and $Q_{l_k}$ are always kept constant. For a generation bus, known as PV bus in which the voltage is constant a new Eq. (3) must be added to the set of Eqs. (1) and (2). As a consequence, the corresponding reactive power $Q_{g_k}$ becomes a new power flow variable in the power flow formulation.

$$V_k^2 = V_{r_k}^2 + V_{m_k}^2 \tag{3}$$

The active power generation at PV buses are here modeled considering the steady state primary frequency regulation mathematical model [15,17,19,20], which is given by:

$$y_k = P_{g_k}-P_{g_k}^{spec} + \frac{1}{R_k}(f-f_0) = 0 \tag{4}$$

where:

| | |
|---|---|
| $P_{g_k}^{spec}$ | Active power injection at bus $k$; |
| $P_{g_k}$ | Active power output of generator $k$; |
| $f$ | System frequency; |
| $f_0$ | System frequency reference; |
| $R_k$ | Generation unit statism. |

It is important to notice that from Eq. (4) for $n_g$ generation buses there will be $n_g + 1$ variables because the system frequency becomes a new variable in this formulation. In the proposed methodology a new Eq. (5) is included in the system of equations, meaning that any bus of the system can be a reference bus. Then a new concept of slack bus is introduced in this formulation being this the main contribution of this paper. Eq. (5) is then converted to rectangular coordinates, as in Eq. (6), as follow:

$$\theta_p-\theta_p^{spec} = 0 \tag{5}$$

$$\arctan\left(\frac{V_{m_p}}{V_{r_p}}\right)-\theta_p^{spec} = 0 \tag{6}$$

As a conclusion, a system having a total of $n$ buses there will be $(2n + 2n_g + 1)$ equations to be solved using the Newton-Raphson method. The corresponding augmented linearized system of Equations, which is solved at each iteration is given by (7) as follows:

$$\begin{bmatrix} \Delta\underline{I_m} \\ \Delta\underline{I_r} \\ \Delta\underline{V} \\ \hline \Delta\underline{y} \\ \Delta\theta \end{bmatrix} = \left[\begin{array}{ccc|cc} \frac{\partial \underline{I_m}}{\partial \underline{V_r}} & \frac{\partial \underline{I_m}}{\partial \underline{V_m}} & \frac{\partial \underline{I_m}}{\partial \underline{Q_g}} & \frac{\partial \underline{I_m}}{\partial \underline{P_g}} & \frac{\partial \underline{I_m}}{\partial f} \\ \frac{\partial \underline{I_r}}{\partial \underline{V_r}} & \frac{\partial \underline{I_r}}{\partial \underline{V_m}} & \frac{\partial \underline{I_r}}{\partial \underline{Q_g}} & \frac{\partial \underline{I_r}}{\partial \underline{P_g}} & \frac{\partial \underline{I_r}}{\partial f} \\ \frac{\partial \underline{V}}{\partial \underline{V_r}} & \frac{\partial \underline{V}}{\partial \underline{V_m}} & \frac{\partial \underline{V}}{\partial \underline{Q_g}} & \frac{\partial \underline{V}}{\partial \underline{P_g}} & \frac{\partial \underline{V}}{\partial f} \\ \hline \frac{\partial \underline{y}}{\partial \underline{V_r}} & \frac{\partial \underline{y}}{\partial \underline{V_m}} & \frac{\partial \underline{y}}{\partial \underline{Q_g}} & \frac{\partial \underline{y}}{\partial \underline{P_g}} & \frac{\partial \underline{y}}{\partial f} \\ \frac{\partial \theta}{\underline{V_r}} & \frac{\partial \theta}{\partial \underline{V_m}} & \frac{\partial \theta}{\partial \underline{Q_g}} & \frac{\partial \theta}{\partial \underline{P_g}} & \frac{\partial \theta}{\partial f} \end{array}\right] \begin{bmatrix} \Delta\underline{V_r} \\ \Delta\underline{V_m} \\ \Delta\underline{Q_g} \\ \hline \Delta\underline{P_g} \\ \Delta f \end{bmatrix} \tag{7}$$

where:

| | |
|---|---|
| $\underline{I_m}$ | Vector of dimension $n$ which contains the imaginary part of current injection Eq. (2); |
| $\underline{I_r}$ | Vector of dimension $n$ which contains the real part of current injection Eq. (1); |
| $\Delta\,\underline{I_m}$ | Vector of dimension $n$ which contains the imaginary part of current injection Eq. (2) residues; |
| $\Delta\,\underline{I_r}$ | Vector of dimension $n$ which contains the real part of current injection Eq. (1) residues; |
| $\underline{V_r}$ | Vector of dimension $n$ which contains the real part of the system voltages; |
| $\underline{V_m}$ | Vector of dimension $n$ which contains the imaginary part of the system voltages; |
| $\Delta\,\underline{V_r}$ | Vector of dimension $n$ which contains the real part of the system voltage increments; |
| $\Delta\,\underline{V_m}$ | Vector of dimension $n$ which contains the imaginary part of the system voltage increments; |
| $\underline{V}$ | Vector of dimension $n_g$ which contains the voltage magnitude Eq. (3) for all generation buses; |
| $\Delta\underline{V}$ | Vector of dimension $n_g$ which contains the voltage Eq. (3) residues for all generation buses; |
| $\underline{Q_g}$ | Vector of dimension $n_g$ which contains the reactive power generation for all generation buses; |
| $\Delta\,\underline{Q_g}$ | Vector of dimension $n_g$ which contains the reactive power generation increments for all generation buses; |
| $\underline{P_g}$ | Vector of dimension $n_g$ which contains the active power generations for all generation buses; |
| $\Delta\,\underline{P_g}$ | Vector of dimension $n_g$ which contains the active power generation increments for all generation buses; |
| $\underline{y}$ | Vector of dimension $n_g$ which contains the primary regulation Eq. (4) for all generating units; |
| $\Delta\underline{y}$ | Vector of dimension $n_g$ which contains the primary regulation Eq. (4) for all generating unit residues. |

The current residues can be calculated as in Eqs. (8) and (9), as follows:

$$\Delta I_{m_k} = \frac{\Delta P_k V_{m_k}-\Delta Q_k V_{r_k}}{V_{r_k}^2 + V_{m_k}^2} \tag{8}$$

$$\Delta I_{r_k} = \frac{\Delta P_k V_{r_k} + \Delta Q_k V_{m_k}}{V_{r_k}^2 + V_{m_k}^2} \tag{9}$$

where:

$$\Delta P_k = P_{g_k}-P_{l_k}-P_k^{calc} \tag{10}$$

$$\Delta Q_k = Q_{g_k}-Q_{l_k}-Q_k^{calc} \tag{11}$$

The calculated active and reactive power injections can be determined as in Eqs. (12) and (13), as follows:

$$P_k^{calc} = V_{r_k}I_{r_k}^{calc} + V_{m_k}I_{m_k}^{calc} \tag{12}$$

$$Q_k^{calc} = V_{m_k}I_{r_k}^{calc}-V_{r_k}I_{m_k}^{calc} \tag{13}$$

The voltage residue for a generation at bus $k$ is obtained from the following Eq. (14):