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Short Communication

New Parseval's inactive-power factor of a two-terminal network

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ABSTRACT

This article is a continuation of the study on a new definition of the inactive-power factor of a two-terminal network. A new definition of an inactive-power measure, which does not depend on frequency, and a new formalized approach to calculate it in a discrete time domain, is presented here. The inactive-power factor has been called Parseval's factor by analogy to a similar coefficient used in harmonic analysis.

1. Introduction

The article proposes a new inactive-power factor definition, which results from the distribution of the load's admittance operator to the product of the self-adjoint and unitary operators.

This distribution is implemented in a discrete time domain with the use of special 'functional digital filters', and works as an alternative to a different distribution to the sum of orthogonal operators.

A useful, previously unknown inactive-power factor results from a comparison of these two distributions. It is obtained as a hyperbolic sine of the skew-Hermitian operator (digital filter).

This indicator can be determined numerically using time sampling of two reciprocal operators: the impedance and admittance operators.

Thanks to the mathematical functional analysis, used in the article, the mentioned power factor has been obtained in a universal form.

This means that it can be used in a discrete-time domain as well as in the corresponding frequency domain.

However, it has been shown that the continuous-time domain cannot be used herein because the required mathematical procedures do not converge.

That is why, for the first time, digital filters with special operational algorithms have been used for this purpose.

The inactive-power factor can be of great practical importance because it is particularly easily and quickly achievable in discrete-time domain using signal processors.

In addition, the possibility to determine this indicator in the frequency domain also provides some valuable information regarding the power quality of the load.

The well known and wildly used reactive-power factors are based on harmonic distributions of signals [3,11–15].

Therefore, they are inevitably dependent on the frequency as well as on supplying-voltage waveform.

The power factor proposed herein is of an objective nature, i.e. it does not depend on the shape of the source signal but only on the admittance operator of the load.

Because of its versatility, it is better than the power factor used so far.

Such a power factor has not yet been published in the scientific literature, so it has a high innovation value and its potential impact in the field is still difficult to estimate.

A deeper comparison of power factors will be introduced later in the text.

2. Basic information about digital filters

A digital filter, which is defined by its weight sequence $\{A_m\}_{m=-\infty}^{\infty}$, converts the input sequence $\{x_m\}_{m=-\infty}^{\infty}$ into an output sequence $\{Ax\}_m$ according to the convolution algorithm [2,5,10]

$$(Ax)_n = \sum_{m=-\infty}^{\infty} A_m x_{n-m} = \left(\left(\sum_{m=-\infty}^{\infty} A_m z^m\right)x\right)_n \tag{1}$$

where z is a unit delay operator:

$$(zx)_n = x_{n-1} \Rightarrow (z^m x)_n = x_{n-m}$$

The two equivalent digital filter representations, one in 'n' (time-domain) and the second in 'z-domain' (operator or complex domain), result from the expression (1):

$$A(z) = \sum_{m=-\infty}^{\infty} A_m z^m, \ z \in \mathbf{C},$$
$$A_n = \frac{1}{2\pi j}\oint_{|z|=1} A(z) z^{-n} d(\ln z) \tag{2}$$

The scalar product in the signals' linear space, as well as in digital-

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filter space, can be introduced as follows:

$$(A, B) = \sum_n A_n B_n. \tag{3}$$

where the summation in (3) is extended to all integers, i.e. $n \in (-\infty, \infty)$.

Adjoint operator to a filter A is called a A^* filter that for any x, y sequences (also called signals) satisfies the equation:

$$(Ax, y) = (x, A^*y). \tag{4}$$

It is not difficult to show that in the n-domain

$$A_n^* = A_{-n} \tag{5}$$

and in z-domain

$$A^*(z) = A(z^{-1}) \tag{6}$$

In both 'n-' and 'z-'domain it holds

– composition (product or sequence) of filters

$$(A, B)_n = \sum_m A_{n-m} B_m. \tag{7}$$

– Borel's convolution theorem:

$$((AB)_n)(z) = \left(\sum_m A_{n-m} B_m\right)(z) = A(z)B(z) \tag{8}$$

$$(A_n B_n)(z) = \frac{1}{2\pi j} \oint_{|w|=1} A(zw^{-1})B(w)d(\ln w) \tag{9}$$

– filters' correlation

$$(AB^*)_n = \sum_m A_{n-m} B_{-m} = \sum_m A_{n+m} B_m \tag{10}$$

– Parseval's identity:

$$(A, B) = \sum_n A_n B_n = [(A_n B_n)(z)]_{z=1} = \frac{1}{2\pi j} \oint_{|z|=1} A(z^{-1})B(z)d(\ln z) \tag{11}$$

In formulas (9) and (11), the integral of a complex-variable function is taken on a unit circle in mathematically positive direction.

Causal filter is such that $A_n = 0$ for $n < 0$ and is **stable** when:

$$\sum_{n=-\infty}^{\infty} |A_n| < \infty \leftrightarrow \forall_{z:|z|\leqslant 1} |A(z)| < \infty$$

3. Polar and orthogonal distribution of a causal filter

Causal filters are digital (digitized) models of an impedance and admittance operators of a two-terminal electrical network (electrical load).

In particular Y (the receiver's admittance operator) has an important application from the theory of electric power quality point of view.

Once digitized, it is identified by Y_n sequence satisfying the causality condition i.e. $Y_n = 0$ for $n < 0$.

Polar decomposition of a causal Y filter is defined as follows:

$$Y = HU \tag{12}$$

where H is self-adjoint (Hermitian) filter:

$$H = H^* \tag{13}$$

and U is so called unitary filter:

$$U^* = U^{-1} \tag{14}$$

which norm is equal to 1

$$\|U\|^2 = (U, U) = 1 \tag{15}$$

The unitary filter has an exponential representation:

$$U = e^{\phi} \doteqdot \sum_{n=0}^{\infty} \frac{\phi^n}{n!} \tag{16}$$

where ϕ is an anti-Hermitian filter i.e.:

$$\phi^* = -\phi$$

Unitary filter has also an **orthogonal distribution** defined as:

$$U = \sum_{n=0}^{\infty} \frac{\phi^{2n}}{(2n)!} + \sum_{n=0}^{\infty} \frac{\phi^{2n+1}}{(2n+1)!} \doteqdot \cosh(\phi) + \sinh(\phi)$$
$$= \frac{1}{2}(U + U^*) + \frac{1}{2}(U - U^*) \tag{17}$$

Using a simple identity of filters scalar product

$$(A, B) = (AB^*, I) = (A^*B, I) \tag{18}$$

where I is Kronecker's unit signal (sequence), such that:

$$I_n = \begin{cases} 1 & for\ n = 0 \\ 0 & for\ n \neq 0 \end{cases}$$

and taking into account that $UU^* = I$ it's easy to show that component filters $\cosh(\phi)$, $\sinh(\phi)$ in (17) are orthogonal, i.e.:

$$(\cosh(\phi), \sinh(\phi)) = 0 \tag{19}$$

and:

$$\|\cosh(\phi)\|^2 + \|\sinh(\phi)\|^2 = 1 \tag{20}$$

$$[\cosh(\phi)]^2 - [\sinh(\phi)]^2 = I \tag{21}$$

The Eq. (21) entitles the $\cosh(\phi)$ and $\sinh(\phi)$ filters to be called 'hyperbolic', because (21) is in fact a 'hyperbolic operational equation'.

Filters H, U, ϕ are non-causative while the Y admittance is a causative filter.

Its full polar and orthogonal distribution can be rewritten in common form

$$Y = He^{\phi} = H\cosh(\phi) + H\sinh(\phi) = G + B \tag{22}$$

where the G, B operators have the following properties:

$G^* = G$ – is Hermitian
$B^* = -B$ – is anti-Hermitian

The energy (or active power) consumed by Y by means of voltage 'v' and current 'i' is calculated as a scalar product (v, i) which yields

$$(Yv, v) = (Gv, v)$$

because

$$(Bv, v) = (B^*v, v) = -(Bv, v)$$

The B operator is thus an inactive component of the Y admittance operator of a two-terminal network.

So the $\|\sinh(\phi)\|^2$ norm can be regarded as inactive-power factor, whereas the $\|\cosh(\phi)\|^2$ norm stands for a numerical active-power factor, respectively.

These indicators are complementary in the unitary Pythagorean theorem (20).

For polar distribution of a causal Y operator it holds that:

$$(Y, Y^{-1}) = (HU, H^{-1}U^*) = (U^2, I)$$

and: