Electrical Power and Energy Systems 101 (2018) 482–490

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Saturable reactor hysteresis model based on Jiles–Atherton formulation for ferroresonance studies

Wenxia Sima[a], Mi Zou[a,b], Ming Yang[a,c,*], Daixiao Peng[a], Yonglai Liu[a]

[a] State Key Laboratory of Power Transmission Equipment & System Security and New Technology, Chongqing University, Chongqing 400044, China
[b] Centre for Applied Power Electronics, Department of Electrical & Computer Engineering, University of Toronto, Toronto, ON M5S 3G4, Canada
[c] Department of Electrical and Computer Engineering, Tandon School of Engineering, New York University, Brooklyn, NY 11201, USA

ARTICLE INFO

Keywords:
Jiles–Atherton model
Reactor
Ferroresonance
Electromagnetic transients

ABSTRACT

Despite remarkable achievements in the field of Jiles–Atherton (JA) hysteresis theory, JA hysteresis modeling remains challenging given that most commercial software still lacks a practical dynamic JA hysteresis reactor model for electromagnetic transients. A novel voltage-driven dynamic flux linkage–current (ψ–i) JA hysteresis reactor in Electromagnetic Transients Program–Alternative Transients Program (EMTP–ATP) is developed in this study. The proposed model is based on flux linkage and current instead of magnetic flux density and magnetic field. Voltage-driven dynamic losses are incorporated into the static ψ–i JA hysteresis model by using Type-94 element in EMTP–ATP. The two proposed models in this paper are validated using current tests under 50 and 150 Hz. The performance of the proposed dynamic Model 1 matches better with experiments than the dynamic Model 2. Ferroresonance tests are carried out to validate the performance of the proposed reactor. The results show that the proposed reactor has a broad application prospect in electromagnetic transient studies.

1. Introduction

Ferroresonance is one of the most common low-frequency electromagnetic transients (EMTs) [1–4]. The occurrence of ferroresonance requires a nonlinear inductance (the saturable iron core of the transformer or the reactor), a capacitance, low power loss condition and a voltage source [5–7]. Because of the modeling complexity and computational burden associated with hysteresis nonlinearities, a simple mathematical expression (e.g., piecewise-linear, exponential and polynomial) is widely used to represent the nonlinear characteristic of the transformer core in most ferroresonance studies [5,8–10]. With the development of transformer design, the width of the transformer iron core hysteresis loop has narrowed significantly. The hysteresis loop approximates to the anhysteretic loop. Thus, the single-valued magnetization characteristics are acceptable for the steady-state power quality studies such as harmonic power flow [11].

However, the anhysteretic approximation has been proven to be inadequate for the study of dynamic, transient and nonsinusoidal power system studies. The operating points of the system are significantly affected by the dynamic excitation of a nonlinear hysteretic core. Thus, the modeling of major and minor hysteresis loop trajectories becomes very important in these studies. For ferroresonance study, the accurate modeling of hysteresis loops is especially significant because the major and minor loops can potentially generate more ferroresonant operating points [6].

Another aspect of hysteresis phenomenon in the analysis of ferroresonance is the inadequate representation of core loss. In most of the existing studies, the core loss is commonly described by a constant resistance or nonlinear resistance [6]. However, the constant or nonlinear resistance cannot accurately represent the core loss because the core loss in the transformer should dynamically decrease as the core excitation level increases [6]. Thus, ferroresonance simulation still remains a challenge due to the accurate modeling of the hysteresis reactor [12–17].

Among the existing literature of hysteresis models, the Stoner–Wolhfarth model, the Jiles–Atherton (JA) model, the Globus model, and the Preisach model are widely reported [18–20]. These models use different theoretical assumptions, so their performance and applicability are also different. The JA model is proved to be best suited to the bulk material and medium ferrites [18]. Besides, the identification of the parameters of JA model requires relatively fewer measurements [19], and the accuracy of this model in EMT simulation has been widely verified [21–27]. Therefore, in this paper, the JA hysteresis model is used to ferroresonance simulation.

Mutual magnetic interaction and domain wall motion are considered in the JA hysteresis model, which also provides mathematical

* Corresponding author at: State Key Laboratory of Power Transmission Equipment & System Security and New Technology, Chongqing University, Chongqing 400044, China.
E-mail address: cqucee@cqu.edu.cn (M. Yang).

https://doi.org/10.1016/j.ijepes.2018.04.003
Received 13 January 2018; Received in revised form 5 March 2018; Accepted 2 April 2018
Available online 13 April 2018

formulas to explain the hysteresis of magnetization M against magnetic field H on the assumption of uniformly impeded domain wall motion [21–23]. The original magnetic flux density and magnetic field (B–H) expressions are used in the majority of the existing applications of the JA model for EMT simulation [24–27].

Basically, the input and output of these models are flux density and magnetic field, respectively. These B–H JA models are inconvenient for EMT study because the input and output in most EMTP–type platforms are node voltage and branch current, respectively. In addition, only a few of the existing studies include eddy current loss and excess loss in their JA models. Thus, their JA models cannot accurately be used to describe the dynamic loss in the transformer core.

A dynamic voltage-driven ψ–i JA reactor is proposed in this paper. The proposed reactor is implemented in EMTP–ATP using the Type-94 component and MODEL language. In the proposed reactor, the traditional JA hysteresis model represented by magnetic flux density and magnetic field are converted to flux linkage and current. The main features of the proposed reactor are:

(1) The JA hysteresis model in the proposed reactor is converted into electrical quantities from magnetic quantities. This feature contributes to the application of the JA hysteresis theory in widely-used EMTP–type platform.
(2) Instead of flux density, the potential difference of the proposed reactor is used to calculate the dynamic losses.
(3) The parameter estimation of the proposed hysteresis reactor depends on the electrical quantities instead of B–H hysteresis loops.
(4) The asymmetric minor hysteresis loops can be obtained using the proposed reactor in ferroresonance simulation.

The rest of this paper is organized as follows: Two types of classical B–H JA hysteresis models are discussed in Section 2. The development of the two static ψ–i JA models and their voltage-driven dynamic representation are discussed in Section 3. The implementation of the proposed reactor using the ψ–i JA model in EMTP–ATP is discussed and a simulation example is provided to show the dynamic losses in Section 4. The ring core current experiments are used to calculate the parameters and compare the two proposed models in Section 5. Ferroresonance is studied to validate that the proposed reactor can work correctly in EMT simulation in Section 6. Lastly, the conclusion is presented in Section 7.

2. Review of the *B–H* JA hysteresis model

The JA model was first proposed in 1986 [21]. Then, various expressions of the static JA model were presented in [22–28]. Among these models, two types of JA hysteresis theory expressions were widely used.

2.1. Classical B–H JA model

The anhysteretic magnetization curve (AMC) of the classical B–H JA model is commonly represented by Langevin function as

$$M_{an}(H_e) = M_s[\coth(H_e/a) - (a/H_e)] \tag{1}$$

where M_s is the saturation magnetization, $H_e = H + \alpha M$ is the effective field, a is the anhysteretic form factor, and α is the interdomain coupling coefficient.

Nevertheless, the main differential equation of JA hysteresis theory was presented differently in [29,30] and even in the original publications of D.C. Jiles. For example, the differential equation in [21] is given as

$$\frac{dM}{dH} = \frac{1}{1+c}\frac{1}{k\delta/\mu_0 - \alpha(M_{an}-M)}(M_{an}-M) + \frac{c}{1+c}\frac{dM_{an}}{dH} \tag{2}$$

where k is the coercive field magnitude, c is the magnetization weighting factor, and δ is a directional parameter and takes the value $+1$ for $dH/dt > 0$ and -1 for $dH/dt < 0$.

In [22], this differential equation is defined as

$$\frac{dM}{dH} = (1-c)\frac{M_{an}-M_{irr}}{k\delta - \alpha(M_{an}-M_{irr})} + c\frac{dM_{an}}{dH}. \tag{3}$$

However, in [23], the differential equation is given by

$$\frac{dM}{dH} = (1-c)\frac{M_{an}-M}{k\delta - \alpha(M_{an}-M)} + c\frac{dM_{an}}{dH}. \tag{4}$$

Given the non-standardized representation of the main JA differential equation, it is briefly derived again in this study as follows:

The original energy balance equation is defined as

$$\mu_0\int M_{an}dH_e = \mu_0\int MdH_e + \mu_0 k\delta\int\frac{dM_{irr}}{dH_e}dH_e. \tag{5}$$

Substituting $dM_{irr} = (dM - cdM_{an})/(1-c)$ in (5) provides

$$\mu_0\int M_{an}dH_e = \mu_0\int MdH_e + \frac{\mu_0 k\delta}{1-c}\int\frac{dM}{dH_e}dH_e - \frac{\mu_0 k\delta c}{1-c}\int\frac{dM_{an}}{dH_e}dH_e. \tag{6}$$

By differentiating in terms of H_e on both sides of (6), (7) can be obtained as follows:

$$\mu_0 M_{an} = \mu_0 M + \frac{\mu_0 k\delta}{1-c}\frac{dM}{dH_e} - \frac{\mu_0 k\delta c}{1-c}\frac{dM_{an}}{dH_e}. \tag{7}$$

With $dH_e = dH + \alpha dM$ and $dB = \mu_0(dH + dM)$, (7) can be rewritten as

$$\mu_0 M_{an}(dH + \alpha dM) = \mu_0 M(dH + \alpha dM) + \frac{\mu_0 k\delta}{1-c}dM - \frac{\mu_0 k\delta c}{1-c}dM_{an}. \tag{8}$$

Then, the following expressions are obtained:

$$\frac{dM}{dH} = \left(M_{an} - M + \frac{k\delta c}{1-c}\frac{dM_{an}}{dH}\right)\Big/\left(\alpha M + \frac{k\delta}{1-c} - \alpha M_{an}\right). \tag{9}$$

By substituting $\frac{dM_{an}}{dH} = \frac{dM_{an}}{dH_e}\left(1 + \alpha\frac{dM}{dH}\right)$ in (9), the main differential equation of static JA model can be obtained as

$$\frac{dM}{dH} = \left(M_{an} - M + \frac{k\delta c}{1-c}\frac{dM_{an}}{dH_e}\right)\Big/\left[\alpha(M - M_{an}) + \frac{k\delta}{1-c}\left(1 - \alpha c\frac{dM_{an}}{dH_e}\right)\right]. \tag{10}$$

2.2. Modified B–H JA model

To provide greater flexibility for obtaining a good overall shape of the simulated hysteresis loop than the Langevin function [25,26], the AMC is modified by

$$M_{an} = M_s\frac{a_1 H_e + H_e^2}{a_3 + a_2 H_e + H_e^2}. \tag{11}$$

(1) and (11) are both subjected to the following properties:

$$\lim_{H_e\to 0}M_{an} = 0, \lim_{H_e\to\infty}M_{an} = M_s, dM_{an}/dH_e \geqslant 0. \tag{12}$$

Thus, (1) and (15) are both proper to represent anhysteretic magnetization characteristics [21].

The main differential equations in [25,26] are modified as

$$\frac{dM}{dH} = \left(c\frac{dM_{an}}{dH_e} + T\right)\Big/\left(1 - \alpha c\frac{dM_{an}}{dH_e}\right) \tag{13}$$

$$\begin{cases} T = \dfrac{M_{an}-M}{\frac{\delta k}{\mu_0} - \frac{\alpha(M_{an}-M)}{1-c}} & \text{for } (M_{an}-M)\delta > 0 \\ T = 0 & \text{for } (M_{an}-M)\delta \leqslant 0 \end{cases} \tag{14}$$

Therefore, among the existing B–H JA hysteresis models, (1) and (10) are one type, and (11) and (13) are another type.