Electrical Power and Energy Systems 101 (2018) 491–499

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

# Contribution of transmission and voltage constraints to the formation of locational marginal prices

Tatiana Vaskovskaya[a,*], Priyanko Guha Thakurta[b], Janusz Bialek[b]

[a] V.A. Trapeznikov of Institute of Control Sciences of Russian Academy of Sciences and National Research University of Moscow Power Engineering Institute, Moscow, Russia

[b] Center for Energy Systems, Skolkovo Institute of Science & Technology, Moscow, Russia

ARTICLE INFO

*Keywords:*
Locational marginal pricing
Optimal power flow
Lagrange multipliers

ABSTRACT

This paper develops analytical formulae for expressing Locational Marginal Prices (LMPs) as the sum of components due to each of transmission and/or voltage binding constraints using full non-linear Alternating Current (AC) power system model. Each LMP component at a node consists of a price-bonding factor (PBF) multiplied by the marginal price of a corresponding marginal node. Hence PBF represents a price linkage between a marginal and a non-marginal node due to a certain system operating constraint. The aim of the methodology is not to determine LMPs but to decompose them in order to identify particular constraints that are affecting the formation, thereby identifying those lines that may need upgrade. The methodology has been tested on 5-node PJM and 2746-node Polish systems.

## Nomenclature

*Sets*

| | |
|---|---|
| $\mathscr{D}$ | binding transmission constraints |
| $\mathscr{N}$ | all nodes in the system |
| $m_P$,($t_P$) | set of real power injection variables at marginal (non-marginal or price-taking) nodes for active power |
| $m_Q$,($t_Q$) | set of reactive power injection variables at marginal (non-marginal or price-taking) nodes for reactive power |
| $m_\theta$,($t_\theta$) | set of voltage angle variables at marginal (non-marginal or price-taking) nodes for active power |
| $m_V$,($t_V$) | set of voltage magnitude variables at marginal (non-marginal or price-taking) nodes for reactive power |
| $m$ | unified set of active and reactive power injection variables $m_P$ and $m_Q$ ($m_P \cup m_Q$) |
| $m_X$ | unified set of voltage angle and magnitude variables $m_\theta$ and $m_V$ ($m_\theta \cup m_V$) |
| $t$ | unified set of active and reactive power injection variables $t_P$ and $t_Q$ ($t_P \cup t_Q$) |
| $t_X$ | unified set of voltage angle and magnitude variables $t_\theta$ and $t_V$ ($t_\theta \cup t_V$) |

*Parameters*

| | |
|---|---|
| $B \in \mathbb{Z}^+$ | number of transmission lines in operation |
| $N \in \mathbb{Z}^+$ | number of nodes in the system |
| $C_{Pg}$ ($C_{Pd}$) $\in \mathbb{R}^N$ | generator (demand) real power bid prices at nodes |
| $C_{Qg}$ ($C_{Qd}$) $\in \mathbb{R}^N$ | generator (demand) reactive power bid prices at nodes |
| $\overline{F} \in \mathbb{R}^B$ | maximum allowable real power flow through lines |
| $\overline{P}_g$ ($\underline{P}_g$) $\in \mathbb{R}^N$ | vector of maximum (minimum) real power generator outputs |
| $\overline{P}_d$ ($\underline{P}_d$,) $\in \mathbb{R}^N$ | vector of maximum (minimum) real power demand outputs |
| $\overline{Q}_g$ ($\underline{Q}_g$) $\in \mathbb{R}^N$ | vector of maximum (minimum) capacitive reactive power generator outputs |
| $\overline{Q}_d$ ($\underline{Q}_d$) $\in \mathbb{R}^N$ | vector of maximum (minimum) inductive reactive power demand and compensator outputs |
| $\overline{V}$ ($\underline{V}$) $\in \mathbb{R}^N$ | vector of maximum (minimum) allowed voltage magnitudes |
| $\overline{\theta}$ ($\underline{\theta}$) $\in \mathbb{R}^N$ | vector of maximum (minimum) allowed voltage angles |
| $\lambda_P$ ($\lambda_Q$) $\in \mathbb{R}^N$ | vector of Lagrange multipliers corresponding to real (reactive) power flow equations |
| $\overline{\mu}_V$ ($\underline{\mu}_V$) $\in \mathbb{R}^N$ | vector of Lagrange multipliers corresponding to maximum (minimum) voltage magnitude constraints |
| $\overline{\mu}_\theta$ ($\underline{\mu}_\theta$) $\in \mathbb{R}^N$ | vector of Lagrange multipliers corresponding to maximum (minimum) voltage angle constraints |
| $\overline{\nu}_g$ ($\underline{\nu}_g$) $\in \mathbb{R}^N$ | vector of Lagrange multipliers corresponding to maximum |

* Corresponding author.
*E-mail address:* vaskovskyata@mpei.ru (T. Vaskovskaya).

https://doi.org/10.1016/j.ijepes.2018.04.004
Received 27 June 2017; Received in revised form 21 February 2018; Accepted 4 April 2018

(minimum) real generation constraints

$\overline{\nu}_d$ ($\underline{\nu}_d$) $\in \mathbb{R}^N$ vector of Lagrange multipliers corresponding to maximum (minimum) real demand constraints

$\overline{\rho}_g$ ($\underline{\rho}_g$) $\in \mathbb{R}^N$ vector of Lagrange multipliers corresponding to maximum (minimum) reactive generation constraints

$\overline{\rho}_d$ ($\underline{\rho}_d$) $\in \mathbb{R}^N$ vector of Lagrange multipliers corresponding to maximum (minimum) reactive demand and compensation constraints

$\sigma \in \mathbb{R}^B$ vector of Lagrange multipliers corresponding to real power flow constraints

*Optimization variables*

$P_g, P_d \in \mathbb{R}^N$ vector of generator (demand) real power injections
$Q_g, Q_d \in \mathbb{R}^N$ vector of generator (demand) reactive power injections
$V \in \mathbb{R}^N$ vector of node voltage magnitudes
$\theta \in \mathbb{R}^N$ vector of node voltage angles

*Other variables*

$F \in \mathbb{R}^B$ vector of real power flow through lines
$P \in \mathbb{R}^N$ vector of net real power injections
$Q \in \mathbb{R}^N$ vector of net reactive power injections

In the subscript of a matrix, an alphabet following a colon (:) implies that all the columns of the matrix designated by the alphabet set are considered, whereas a colon (:) following an alphabet implies that all the rows of the matrix designated by the alphabet set are considered.

## 1. Introduction

In order to increase competition among different stakeholders electricity sector has been decentralized in many parts of the world leading to rapid changes in industry structure and policies. Nodal pricing of electricity has gained an increased importance in order to utilize generation and transmission resources efficiently in a competitive environment and manage system congestion, especially through the use of locational marginal pricing (LMP) [1,2]. LMP is one of the most powerful means of congestion management in many electricity markets worldwide, such as in the United States, Australia, Russia, etc.

The LMP at a particular node can be defined as the marginal cost that would be incurred to supply an additional increase of a load at that particular node while respecting all the security limits of the system. The values of LMPs differ among the nodes due to the system hitting its security limits and/or transmission losses.

LMP decomposition and nodal pricing were first introduced in the eighties [3–5]. Such a decomposition approach is termed classical, and decomposes each LMP into energy, loss and congestion components. LMP decomposition is based either on linearized[1] or non-linearized[2] optimization, and can either be reference node independent or dependent. The aim of calculating components is twofold. Firstly, the components are used to calculate LMPs [6]. Secondly, components are calculated for the purpose of financial instruments such as Financial Transmission Rights (FTRs) [7] and Loss Hedging Rights (LHRs) [8]. LMPs represent price signals and their components provide transparency of pricing. LMP components produce useful information for promoting investment in transmission system [9] and congestion management [10]. Particularly, they are used to introduce congested zones [11], which are necessary, for example, for calculating certain power market indices in market monitoring issues [12].

Expression for reference node dependent components of LMPs are derived in [13–15]. LMP decomposition based on distributed slack is proposed in [16]. However, the reference node dependent approach (which also includes randomly selected distributed slack) has severe limitations [17,18]. Such limitations are addressed with reference node independent decomposition in [19–23] considering DCOPF and ACOPF respectively. However, different results may be produced by DCOPF and ACOPF, as shown in [24]. In [19,23], the congestion component is obtained in a reverse manner based on KKT conditions and marginal losses. Approaches introduced in [20–22] do not work with non-linearized version of economic dispatch. Additional components are also included in the LMP decomposition such as ancillary services like spinning reserve, voltage and security controls in [25] and additional risk component in [26]. Quadratic target function is studied in [27] in which a connection between cost function and LMPs is proposed for LMP decomposition. Study on dual space of economic dispatch to calculate LMPs is performed in [28]. An in-depth analysis of orthogonal space of Lagrangian multipliers is performed in [29,30]. The energy reference is chosen based on an optimization problem resulting in participation factors, the target function being the welfare of FTR payments [31,32].

The above-mentioned literature does not emphasize the contribution of each binding constraint with respect to the marginal nodes of the system on the formation of LMP at a node, that is, the response of the marginal nodes to the system binding constraints is not captured. Hence, this paper attempts to measure such responses and bridge the gap between the system binding constraints and power flow that generates LMPs in the system, that is to introduce a physical sense to LMPs and their components. The proposed methodology is based on power flow comprising of marginal nodes in the system.[3]

The specific contribution of this paper is the development of a decomposition technique to calculate LMPs in terms of influences of each binding transmission and/or voltage constraints for full AC OPF environment.[4] The developed methodology works also for linearized optimization and expresses LMP at a given node as a linear combination of weighted bids at marginal generators (i.e. generators that do not operate at their limits). The weights are termed as price-bonding factors (PBFs) in this paper and they define a price linkage between a marginal node and a non-marginal node with respect to a certain system operating constraint.

The decomposed PBFs give a signal of sharing costs among the market participants when e.g. a particular constrained line is reinforced. They can also help monitoring electrical energy markets and identifying zones for which market power indices are introduced [33].

The paper is organized as follows. Section 2 gives a brief overview of the mathematical background behind LMPs in a system. The proposed methodology to decompose LMPs in this paper is shown in Section 3. The methodology is applied on two test systems, the results of which are shown in Section 4. Finally, Section 5 draws the conclusions of the paper.

## 2. Mathematical background behind LMPs

This section reviews the mathematics behind LMP decomposition and is required to understand the proposed methodology of this paper. The basic security-constrained economic dispatch can be formulated as:

$$\max \quad C_{Pd}^{\mathrm{T}} P_d + C_{Qd}^{\mathrm{T}} Q_d - C_{Pg}^{\mathrm{T}} P_g - C_{Qg}^{\mathrm{T}} Q_g \tag{1}$$

[1] Direct Current Optimal Power Flow (DCOPF).
[2] Alternating Current Optimal Power Flow (ACOPF).
[3] It can be applied only after solving the economic dispatch problem.
[4] In some countries, such as in Russian Federation, the wholesale power market utilizes such a model both for day-ahead and intra-day markets.