Induction machine model with space harmonics for fault diagnosis based on the convolution theorem

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A B S T R A C T

Fault diagnosis of induction machines (IMs) requires a fast model of the machine, for adjusting fault thresholds in data-driven diagnostic methods, for computing the residuals in model-driven diagnostic systems, or for training autonomous expert systems. Due to the interaction between time and space harmonics under faulty conditions, this model must simulate very accurately the space harmonics of the air gap magnetomotive force (MMF) generated by the machine's windings. But the computation of the phases' inductances, taking into account the spatial harmonics of the MMF, for every angular position of the rotor, and under non-symmetrical, faulty conditions, is a time-consuming task in IMs' models. In this paper, a very fast method for obtaining the inductances of rotating electrical machines is proposed, based on a single discrete circular convolution. With the proposed approach, the mutual inductances of two phases, taking into account the spatial harmonics of the air gap MMF, are calculated for every relative angular position using a single equation, solved with the fast Fourier transform (FFT). Asymmetrical winding distributions, and the linear rise of the air gap MMF across skewed slots are easily modeled without increasing the computation time. The proposed method is introduced theoretically and validated with an experimental test-bed using commercial induction motors with forced broken bars faults.

1. Introduction

Induction machines (IMs) play a key role in modern industry [1], either as motors, such as squirrel cage induction motors (SCIM), or as generators [2], such as doubly fed induction generators (DFIG). For this reason, fault diagnosis of IMs has received an extensive research effort in recent years [3–6]. The development and fine-tuning of fault diagnostic methods, specially when applied to IMs working in transient conditions [7,8], requires a fast model of the machine under faulty conditions [9]. Such a model is necessary to identify the type and the degree of the fault [10,11], to train multi-agent systems to classify induction motor faults [12], to develop vector classifiers [13], to define and compare different fault indexes [14], to estimate the on-line motor parameters [15], or to design and maintain the motor drives [16]. The availability of this type of models allows to reduce the number of destructive tests needed to validate new diagnostic techniques, and to test fault diagnostic techniques implemented in embedded devices such as field programmable gate arrays (FPGAs) [17] or digital signal processors (DSPs) [18]. These models are also very helpful for a better understanding of the observed phenomena [19], which can lead to the

development of new diagnostic techniques. An extensive review of the mathematical models that have been used to simulate induction machines under faulty conditions is provided in [20].

Transient analysis of rotating electrical machines can be performed using the well established d-q model [21,22]. This model neglects the harmonic contents generated by phase windings, but it is simple enough to allow its implementation in fast FPGAs, making it suitable for hardware-in-the-loop (HIL) tests, as in [23], or for building a Matlab/Simulink model, as in [21,24]. But, from a diagnostic point of view, the spatial harmonics cannot be neglected. The interaction of time and space harmonics [25] results in harmonic fluxes, and therefore torque pulsations will be induced on the machine shaft. Detection of the failure modes under the conditions in which both the phase currents and the phase voltages are not sinusoidal and torque pulsations are present on the shaft is a delicate task [26]. Therefore, a machine model which includes the effect of spatial harmonics is required. Such a model was presented in [27]. Afterwards, using the multiple coupled circuit model (MCCM) approach, [28,29], the model of a symmetrical, general *m-n* induction machine was established based on the phase self and mutual inductances, derived on harmonic bases. The accuracy of the analysis

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depends on the number of harmonics included in this calculation. Relying in [30], the winding function approach (WFA) for the calculation of machine inductances, taking into account the space harmonics, was presented in [31]. The WFA approach has been applied to generate models of squirrel cage induction machines in [32], and wound-rotor machines in [33,34], including the effect of slot skewing in [35], slot openings in [33], interbar currents in [36], the geometry of the slots in [37], saturation in [38] and deep bar dependent inductances in [39]. WFA models have been used extensively for fault diagnosis of machines different types of fault: eccentricity in [34], with simultaneous presence of eccentricity and broken bars in [40], turn-to-turn short-circuit faults in stator windings in [41], and bearing faults in [42]. Following a different approach than WFA, other analytical methods such as the generalized harmonic analysis model [43], the Concordia transformations [44], the use of natural variables [45], the voltage-behind-reactance formulation [46], or the magnetic equivalent circuit (MEC) [47–49] have been proposed in the technical literature.

Nevertheless, non-linearities and non-idealities, which cannot be properly modeled using an analytical approach, may also have a strong influence on the suitability of a machine's model for diagnostic purposes. Other techniques that can take them into account, such as the bond graph model of an induction motor in [50], have been proposed. Among them, in recent years, finite elements (FEM) models have been extensively used for machine simulation [51]. For example, in [52], a 2D magnetic and 3D mechanical coupled FEM model for the study of the dynamic vibrations in the stator of induction motors is presented, and in [53] the influence of the ventilating holes in the condition monitoring of the machine has been addressed using a FEM model.

Although the more comprehensive approaches, such as FEM, often produce better results in terms of accuracy, they require a significant computational capacity. Differences of more than 5 h for a FEM analysis (FEA) versus 10 min for the same analysis carried out via a time harmonic method have been reported in [54]. In spite of the tremendous improvements of computers speed, the computational effort required to complete FEM evaluation is significant even with modern processing power [55]. Accurate computation of machine quantities with a very high spatial frequency, such as the torque in fractional-slot concentrated windings, requires a high number of mesh elements along the air gap, causing the computational time to become excessive [56]. The simulation results of the torque and stator inductances obtained by the WFA have been compared with the results obtained by two-dimensional FEA in [57], and the conclusion is that both methods give approximately the same results but require quite different computation times (8 h for the FEM analysis vs. 1 min for the WFA one). The use of MEC approach has been proposed in [49] based on the advantage that it includes spatial dependencies as in the case of FEM analysis, but is computationally less intense. The same argument is used by [58,59] to choose the analytical computation of the magnetic field in the air gap instead of a FEM based approach. To reduce the computation time of FEM models, a combined analytical-FEM approach is proposed in [60], the use the machine inductances obtained by FEM in a lumped parameter model is proposed in [61], and a field reconstruction technique based on the resolution of a few FEM models and their linear superposition is presented in [62,63]. The savings in computational effort that circuital methods achieve are essential in situations where a large number of studies are required, such as in optimization of the motor control [45], the analysis of inverter-motor interaction [64], or in population-based automated design techniques [58].

In this context, this paper proposes a new, and very fast procedure, to compute the motor inductances that appear in the motor equations using the coupled circuit approach. The assumptions of this new method are the same than in WFA (magnetic saturation is neglected, and only the radial component of the air-gap magnetic field is considered). In spite of being a method of general validity for calculating the inductances of rotating electrical machines, WFA has some drawbacks: to account for coil pitch, slot skewing or the rise of the air gap

MMF across the slot, different winding functions must be used in each case. Besides, the winding function of a phase must be computed using both the winding functions of the coils that constitute the phase and the coils distribution. Finally, the winding functions must be integrated to obtain the phase inductances, and complex integrals must be solved in this process, which may be very cumbersome in the case of arbitrary winding distributions. As it is stated in [31], this task typically consumes a high amount of time, so that only discrete curves of inductance versus rotor position are calculated and linear interpolation is applied at intermediate rotor positions. In this paper, a completely different way of addressing this problem is undertaken, presenting a new method that is characterized by the following main points:

1. The conductor, instead of the coil, is used as the basic winding unit, which simplifies the modeling of arbitrarily complex windings layouts.
2. The yoke flux of the conductor, instead of the MMF of the coil, is used as the characteristic function of the system, which avoids the MMF to flux transformations.
3. A single circular convolution, instead of the integrals of the winding functions for every rotor position, is used to obtain the winding inductances.

These features represent a significant improvement compared with the WFA method, which can be summarized as follows:

1. Instead of using a different winding function for each type of the coils that constitute the winding, a single function is needed, the yoke flux of a conductor placed at the origin and fed with a unit current.
2. The winding functions represent the MMF generated by each type of coils, so to obtain the flux generated by the coil a further computation is needed. This is avoided in the proposed method, whose characteristic function is the flux generated by a single conductor, instead of its MMF.
3. The winding functions must be integrated for obtaining the flux generated and the flux linked by each phase, and this process must be repeated for each different position of the rotor when computing the mutual inductances between rotor and stator phases. These integrals are replaced in the proposed approach by a single element-wise product of three vectors in the frequency domain: two vectors defining the conductors layout of the phases whose mutual inductance is being calculated, and the vector with the yoke flux generated by a single conductor.

The results obtained with the proposed method are exactly the same as those obtained with the WFA method (the assumptions are the same), but using a process that reduces to just one equation the set equations needed to obtain the mutual inductance of two phases for each relative angular position (which, in the case of a stator and a rotor phases, varies with the rotor position). And this equation is solved in a very fast way thanks to the use of the FFT. It is a very general and rather unconscious assumption to associate the FFT exclusively to signal analysis in the time domain. Nevertheless, it proves to be also extremely powerful when applied to the treatment of quantities in the spatial domain, such as the air gap MMF and the yoke flux. The expressions derived for these quantities have a mathematical structure analogous to the ones found in the analysis of many time signals, so that the tools used in this field, as the FFT, can be successfully applied to compute phase inductances in a very fast way. In the experimental section of this paper, the time needed to obtain the winding inductances of the motor in Appendix A has been reduced from 7.571 to 0.261 s, giving exactly the same results.

The considerations explained above (that is, applying convolution and FFT to suitable machine space quantities) constitute the driving idea behind the new method proposed in this paper, whose feasibility