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Integration of voltage dependent power injections of distributed generators into the power flow by using a damped Newton method

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ABSTRACT

Voltage dependent active and reactive power injections are used to improve the grid integration of distributed generators, such as photovoltaic systems. In state-of-the-art literature these power injections are considered during power flow calculations via an external loop, which requires to perform multiple (up to ten or more) power flow calculations in order to obtain one static operation point. In this paper it is shown how to integrate voltage dependent power injections into the nonlinear grid equations in order to solve for a static working point by means of performing one Newton power flow calculation. That makes the external loop unnecessary and save computational time. Furthermore, it is shown for a two-bus system that the integration of voltage dependent power injections into the nonlinear grid equations can exhibit ill-conditioned power mismatch functions. Therefore, a damped Newton method is used in order to avoid numerical oscillations. Simulations of the IEEE 118-bus test system and a real German 234-bus test system show that the damped method converges faster and is computationally more efficient than the external approach currently used in literature and state-of-the-art simulation tools in power systems. In addition, the implemented power flow algorithm is validated in the laboratory for a two-bus system and a numerical example of a low-voltage feeder is given.

1. Introduction

In recent years, the penetration of medium and low voltage grids with inverter coupled distributed generators (DGs) has increased significantly and is expected to keep growing [1,2]. The intermittency and high amount of active power infeed of DGs can cause voltage fluctuations [3]. In order to adhere to the respective voltage quality standards like IEC 60038 [4] or EN 50160 [5] DGs can be equipped with voltage dependent active and reactive power injections, also called Volt/VAR control [6], volt/watt control or local voltage control [7,8]. The introduction of voltage dependent power injections is, amongst others, motivated by the standard IEEE 1547 [9] in order to mitigate voltage rises caused by DGs [10–13]. In order to conduct realistic power system studies like grid planning it is necessary to take voltage dependent power injections of DGs into account during the calculation of static operation points, namely during the power flow.

The state-of-the-art method (to take voltage dependent power injections of DGs into account during the calculation of a static operation point) consists of performing multiple power flows. This is done by updating the injected active and reactive power after each power flow iteratively until a convergence is reached [10,11,14]. We refer to this solution method as *external algorithm* in order to simplify the further explanations given in this paper.

The goal of this paper is to show how voltage dependent power injections of DGs can be integrated into the Newton (or Newton-Raphson) power flow algorithm [15]. This is done by incorporating the voltage dependent power injections into the nonlinear grid equations and using a damped Newton method presented in [16]. We refer to this solution method as *internal algorithm* in order to distinguish between the proposed solution method and the state-of-the art method. The damped (underrelaxed, modified) method used in this paper is presented in [16] under the name bisection method. It is based on famous work done in the 19th and 20th century, e.g. [17–19]. A recent overview can be found in [20]. The main contribution of this paper is the application of the well-known damped Netwon method to the problem of incorporating voltage dependent power injections of DGs into the power flow problem. Another contribution of this paper is to show that voltage dependent power injections can cause ill-conditioned power mismatch functions, which is not very common for power systems according to [16]. Due to this, the use of the damped Newton method is required in

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order to obtain a converging power flow. Furthermore, the *internal algorithm* is compared to the *external algorithm*. We conduct this comparison to show the benefits of our approach compared to the state-of-the-art method. Also, this comparison is used to validate the computation results of the *internal algorithm*. Both algorithms are implemented in MATLAB [21], they are stand-alone and can be used independently of each other. We use an ill-conditioned two-bus system to show that the *internal algorithm* can have superior convergence properties compared to the *external algorithm*. Furthermore, the *internal algorithm* is validated for the case of a two-bus system with a laboratory setup. We apply the proposed *internal algorithm* to the IEEE 118-bus test system and a real German 234-bus grid and demonstrate that for these cases, the *internal algorithm* is computationally faster than the *external algorithm*.

The paper is structured as follows: We outline the modeling of voltage dependent power injection strategies in Section 2. In Section 3 we describe the state-of-the-art *external algorithm* and describe the *internal algorithm*. Test cases are presented in Section 4. In Section 5 a numerical example of a 19 node low-voltage test feeder with five photovoltaic systems is presented. In Section 6 we summarize the findings of the paper and highlight the advantages of the *internal algorithm*.

2. Modeling of voltage dependent power injection

Power injection methods of DGs can be categorized into two groups, as shown in Table 1. Group (a) is characterized by a reactive power injection that is either constant or depends on the injected active power. Here, φ is the angle of the injected complex power and $\cos(\varphi)$ is called power factor. DGs which employ one of these strategies are treated as constant power loads (i.e. P,Q constant) during a power flow. Therefore, no further elaboration is needed for this group from an algorithmic point of view as these are not voltage dependent. Injection methods from control group (a) are practically used by transmission and distribution system operators and are specified in technical standards, such as VDE-AR-N 4120 [22].

Group (b) is characterized by a voltage dependent injection of active and/or reactive power. To consider this group within a power flow calculation, the voltage dependency has to be taken into account. The voltage dependency can be described with a piecewise function $f(V)$ as shown in Fig. 1. This function represents a unified approach for both voltage dependent active and reactive power injection. The maximum active and reactive power, depicted as y_3, is the maximum possible active and reactive power outputs of the DG in the current operating state. The piecewise function $f(V)$ can be described as.

$$f(V) = \begin{cases} y_1, & V \leqslant V_1 \\ y_1 + \gamma_1 \cdot (V - V_1), & V_1 < V \leqslant V_2 \\ y_2, & V_2 < V \leqslant V_3 \\ y_2 + \gamma_2 \cdot (V - V_3), & V_3 < V \leqslant V_4 \\ y_3, & V_4 < V \end{cases} \tag{1}$$

Table 1
Voltage dependent power injection for distributed generators.

Group	Description	Injection method
(a)	Reactive power injection is constant or depends on the injected active power.	$Q_{\text{const.}}$, $\cos(\varphi)_{\text{const.}}$, $\cos(\varphi)(P)$
(b)	Active and/or reactive power injection depend on the bus voltage magnitude	$P(V)$, $Q(V)$

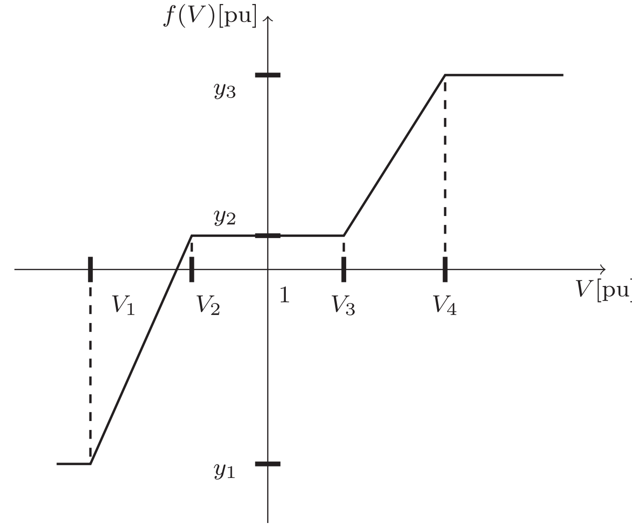

Fig. 1. Piecewise function $f(V)$ for $P(V)$ and $Q(V)$ voltage dependent power injection. This is a generalized depiction of a characteristic. Positive and negative slopes are possible.

where the gradients are $\gamma_1 = \frac{y_2 - y_1}{V_2 - V_1}$ and $\gamma_2 = \frac{y_3 - y_2}{V_4 - V_3}$.

The method proposed in this paper requires to calculate the derivative of (1). The derivative of (1) with respect to the voltage magnitude is a piecewise constant function $g(V)$ with

$$g(V) = \frac{\partial f}{\partial V} = \begin{cases} 0, & V < V_1 \\ \gamma_1, & V_1 < V < V_2 \\ 0, & V_2 < V < V_3 \\ \gamma_2, & V_3 < V < V_4 \\ 0, & V_4 < V. \end{cases} \tag{2}$$

The derivative $g(V)$ is discontinuous and not defined at the points V_1,V_2,V_3,V_4. To overcome this issue, a smoothing function will be introduced in Section 2.1.

By choosing appropriate parameters V_1,V_2,V_3,V_4,y_1,y_2,y_3 the reactive and active power characteristics can be expressed by different realizations of $f(V)$. It is:

$$P(V) = f_{\text{P}}(V) \tag{3a}$$

$$Q(V) = f_{\text{Q}}(V). \tag{3b}$$

The choice of the parameters is typically done by the grid operator based on internal grid planning and operation guidelines. These guidelines may vary for different operators as well as countries.

2.1. Smoothing function

To avoid the issue of the discontinuity of the derivative $g(V)$, the following smoothing function for $f(V)$ is used

$$\begin{aligned} f_s(V) = y_1 & \\ + \tfrac{\gamma_1}{k} & [\ln(1 + e^{k(V - V_1)}) - \ln(1 + e^{k(V - V_2)})] \\ + \tfrac{\gamma_2}{k} & [\ln(1 + e^{k(V - V_3)}) - \ln(1 + e^{k(V - V_4)})] \end{aligned} \tag{4}$$

where the difference between the smoothing function and the original function is reduced by increasing values of k. The derivative is

$$\begin{aligned} g_s(V) = {} & \gamma_1 [\text{logsig}(k(V - V_1)) - \text{logsig}(k(V - V_2))] \\ & + \gamma_2 [\text{logsig}(k(V - V_3)) - \text{logsig}(k(V - V_4))] \end{aligned} \tag{5}$$

with $\text{logsig}(x) = \frac{1}{1 + e^{-x}}$ being the log-sigmoid function.

Fig. 2 shows the original piecewise linear function f from (1) and its derivative g from (2) together with the smoothing functions f_s from (4) and its derivatives g_s from (5) for different values of k. The parameters