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Logically constrained optimal power flow: Solver-based mixed-integer nonlinear programming model

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ABSTRACT

There is increasing evidence of the shortage of solver-based models for solving logically-constrained AC optimal power flow problem (LCOPF). Although in the literature the heuristic-based models have been widely used to handle the LCOPF problems with logical terms such as conditional statements, logical-and, logical-or, etc., their requirement of several trials and adjustments plagues finding a trustworthy solution. On the other hand, a well-defined solver-based model is of much interest in practice, due to rapidity and precision in finding an optimal solution. To remedy this shortcoming, in this paper we provide a solver-friendly procedure to recast the logical constraints to solver-based mixed-integer nonlinear programming (MINLP) terms. We specifically investigate the recasting of logical constraints into the terms of the objective function, so it facilitates the pre-solving and probing techniques of commercial solvers and consequently results in a higher computational efficiency. By applying this recast method to the problem, two sub-power- and sub-function-based MINLP models, namely SP-MINLP and SF-MINLP, respectively, are proposed. Results not only show the superiority of the proposed models in finding a better optimal solution, compared to the existing approaches in the literature, but also the effectiveness and computational tractability in solving large-scale power systems under different configurations.

1. Introduction

Logical constraints, which are one particular kind of discrete or numerical constraints such as logical-and, logical-or, negation, and conditional statements, are considered as the nature of most practical optimization problems, and the practical power systems are no exception. Although the logical constraints exist in most decision-making problems of power systems, due to disjoint functioning regions of generating units, more often than not, for the sake of simplicity and computational tractability, these constraints are neglected. This may facilitate finding an optimal solution, however, on the other hand, an accurate model should embody all operational constraints, otherwise, it may lead to a solution with an unsatisfactory outcome. Thus, an appropriate model or tradeoff, between model accuracy and computational efficiency, should be investigated to counteract the aforementioned drawback.

The AC optimal power flow (ACOPF) problem, even in the theoretical studies, is a highly nonlinear problem, due to active and reactive power flow constraints [1], and considering logical constraints makes it even a more complex and highly nonconvex-nonlinear problem. On the other hand, to have a more practical model, the valve-point effect should be considered [2,3], and this exponentially increases the degree of nonlinearity of the problem. Moreover, considering shunt VAr compensator and more specifically thyristor controlled series capacitor (TCSC) and thyristor controlled phase shifter (TCPS) play a crucial role in practical power system operation and planning problems, by improving the efficiency, voltage fluctuations, and loadability. In planning-based problems, the optimal siting and sizing of FACTS devices are taken into account [4,5], while in operating-based problems, the adjustment of these devices, which have already been optimally placed and sized, is considered [6–8]. Incorporating such devices with high-nonlinearity characteristics besides the integer variables of logical constraints results in a complex mixed-integer nonlinear programming problem. The price to be paid for considering the logical constraints and the flexible AC transmission systems (FACTS) is a dramatic increase in the degree of computational complexity, which if handled without care may lead to intractability. This is one of the main motivations of widely using heuristic-based approaches to solve the practical OPF-based problems [9–12]. These approaches may work well in finding an optimal solution for specific systems or models; however, finding an acceptable solution for other systems and models, especially when logical constraints are taken into account, may require major modifications and adjustments. On the other hand, the most successful approaches, among others, to solve OPF problems such as interior point method

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Nomenclature

(a) Indices

- i,j bus indices
- k index for disjoint operating zones
- ij index for the transmission line or device between bus i and j
- d index for direct power flow
- r index for reverse power flow

(b) Sets

- Ω_b set of buses, $\{1,2,\ldots,N_b\}$
- Ω_g set of generating units, $\{1,2,\ldots,N_g\}$, $\Omega_g \subseteq \Omega_b$
- Ω_l set of transmission elements, $\{1,2,\ldots,N_l\}$

(c) Variables and Functions

- $F_i(\cdot)$ fuel cost function of unit i
- fl_{ij} power flow at branch ij
- n_i nonnegative integer decision making variable for shunt VAR compensator at bus i
- n_{ij} nonnegative integer decision making variable for LTCT at branch ij
- P_{g_i} active power generation of unit i
- P_{ik} active power corresponding to the operating zone k of unit i; used in MINLP models
- p_{ij}^d/p_{ij}^r direct/reverse active power between bus i and bus j of branch ij
- q_{ij}^d/q_{ij}^r direct/reverse reactive power between bus i and bus j of branch ij
- Q_{c_i} shunt VAR compensation of bus i
- Q_{g_i} reactive power generation of unit i
- tp_{ij} transformer tap of branch ij
- u_{ik} binary decision making variables of unit i and operating zone k
- v_i voltage magnitude at bus i
- x_{ij}^c reactance of TCSC at branch ij
- δ_i voltage angle of bus i
- θ_{ij} voltage angle difference between bus i and j, $\theta_{ij} = \delta_i - \delta_j$.
- φ_{ij} phase shift angle of TCPS at branch ij

(d) Constants

- a_i,b_i,c_i cost coefficients of unit i
- b_{ij}^{ch} charging susceptance of branch ij
- b_i^{sh} shunt susceptance of bus i (℧)
- b_{ij} susceptance of branch ij (℧)
- e_i,f_i valve-point cost coefficients of unit i
- $\overline{fl}_{ij}$ maximum power flow of branch ij
- g_{ij} conductance of branch ij (Ω)
- g_i^{sh} shunt conductance of bus i (Ω)
- P_{D_i} active power demand at bus i
- $\underline{P}_{g_i}$, $\overline{P}_{g_i}$ minimum and maximum active power generation limits of unit i, respectively
- $\underline{P}_{g_{ik}}$, $\overline{P}_{g_{ik}}$ minimum and maximum active power limits correspond to operating zone k of unit i, respectively
- $\underline{P}_{ik}$, $\overline{P}_{ik}$ minimum and maximum active power limits correspond to operating zone k of unit i, respectively; used in MINLP models
- Q_{D_i} reactive power demand at bus i
- $\underline{Q}_{C_i}$, $\overline{Q}_{C_i}$ upper and lower limits of shunt VAR compensator at bus i, respectively
- $\underline{Q}_{g_i}$, $\overline{Q}_{g_i}$ minimum and maximum reactive power generation limits of unit i, respectively
- r_{ij} resistance of branch ij (Ω)
- $\underline{tp}_{ij}$, $\overline{tp}_{ji}$ minimum and maximum limits of transformer tap of branch ij, respectively
- $\underline{v}_i$, $\overline{v}_i$ minimum and maximum voltage magnitude limits of bus i, respectively
- x_{ij} reactance of branch ij (Ω)
- $\underline{x}_{ij}^c$, $\overline{x}_{ij}^c$ minimum and maximum reactance of TCSC at branch ij
- z_i number of operating zones for unit i
- τ_i predefined step size for shunt VAR compensator at bus i
- τ_{ij} predefined step size for LTCT at branch ij
- $\underline{\varphi}_{ij}$, $\overline{\varphi}_{ij}$ minimum and maximum limits of phase shift angle of TCPS at branch ij

(IPM) [13], primal–dual IPM [14], predictor corrector IPM (PCIPM) [15], decomposed PCIPM [16], trust region and IPM [17], which are known as the standard methods to solve OPF problems, may properly find an optimal solution for the conventional OPF problems, but when applied to OPF problems with logical constraints and FACTS devices (with discrete decision variables), the reliability of them should be seriously questioned [18]. This shows the exigency of proposing a trustworthy model for logical constrained problems.

Until now, to the best of our knowledge, for the logically constrained ACOPF (LCOPF) problems (with or without considering VAr compensators) there is no solver-based model, although, for the logically constrained economic dispatch (LCED) problem, which is a simplified OPF problem, several solver-based models have been proposed. First, in [19], a mixed-integer quadratic programming (MIQP) model has been proposed, and later, in [20] and [21], the authors have developed the MIQP model by proposing a big-M based MIQP (M-MIQP) model and an unambiguous distance-based MIQP (UDB-MIQP) model, respectively. These models could obtain the global optimal solution of ED problems; however, their incapability in dealing with non-smooth and nonlinear terms is still an undeniable shortcoming that prevents them to be applied to LC-ACOPF problems. In order to consider the nonlinear terms such as transmission losses, first, in [22], and later, in [23], a novel transformation has been introduced. Such transformation may cause significant difficulties for the commercial solvers since (a) it results in non-constant upper and lower limits, and (b) the operation of a unit in only one operating zone is guaranteed by forcing the product of two continuous variables, correspond with two different operating zones of that unit, equals to zero, which is a very-hard equality constraint and causes severe difficulties for commercial nonlinear solvers. Therefore, to deal with this problem, in [22], a semidefinite approach, and in [23], a decomposition technique has been used. Although the aforementioned models are not capable of solving practical-constrained models (either complex ED or OPF problems), they have brought new insights into this area of research by showing the importance of solver-based models. Even in some existing linear models for ACOPF problems, [24] and [25], due to the complexity of linearization that highly depends on the approximation techniques, the logical constraints have been neglected. Therefore, the main motivations of proposing the solver-based MINLP models that may fill the existing gap in this area of research can be summarized as (a) the popularity and efficient outcomes of solver-based models in other areas, and (b) the lack of an efficient solver-based model for LCOPF-based problems. Accordingly, the contributions of this paper are threefold:

(1) A transformation of logical characteristics to mixed-integer nonlinear terms by recasting them to the objective function as: