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Estimating the saddle-node bifurcation point of static power systems using the holomorphic embedding method

Shruti D. Rao [a,*], Daniel J. Tylavsky [a], Yang Feng [b]

[a] School of Electrical Computer and Energy Engineering, Arizona State University, Tempe, AZ 85287, USA
[b] Siemens Industry, Inc., RC-US EM SG PTI SW ARC, 4920 Westway Park Boulevard, Suite 150, Houston, TX 77041, USA

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ABSTRACT

Voltage stability studies have been progressively gaining importance in the power engineering community. Predicting the saddle-node bifurcation point (SNBP) of a power system has become more critical as the power-system loading has increased in many places without a concomitant increase in transmission resources. Since a Newton–Raphson power-flow method is inherently unstable near the SNBP, adaptations such as continuation methods have been used as stabilizers. A new class of nonlinear equation solvers known as the holomorphic embedding method (HEM) is theoretically guaranteed to find the high-voltage solution to the power-flow problem, if one exists, up to the SNBP, provided sufficient precision is used and the conditions of Stahl's theorem are satisfied by the equation set. In this paper, four different HEM-based methods to estimate the saddle-node bifurcation point of a power system, are proposed and compared in terms of accuracy as well as computational efficiency.

Introduction

Because of the difficulty of siting transmission lines, utilities are often forced to serve increased electric power demand, without a concomitant expansion of infrastructure. This can lead to the system being operated closer to its saddle-node bifurcation point (SNBP) and therefore closer to voltage collapse than desired. There are many examples of black-outs occurring because of a slow reduction in voltage magnitudes at buses over a time scale of a few minutes to hours followed by a sudden sharp fall in the voltage magnitudes, e.g., [1]. One goal of a voltage stability study is to determine the voltage stability margin, e.g., the amount of real and/or reactive power that can be added before the system experiences voltage collapse, with the distance to the SNBP being a quick indicator of stability margin.

Significant work has been done to analyze the voltage stability of systems. It has been shown that no dynamics are required to be modeled in order to obtain the SNBP of a system and that the small signal voltage stability limit depends only on the steady-state characteristics of the system [2–4]. New techniques of analyzing the voltage stability in steady-state systems while considering system limits such as generator reactive power limits, voltage magnitude constraints, and AVR constraints have been presented in numerous papers, ([5–13] to cite only a few.) Optimization algorithms such as the genetic algorithm, particle swarm optimization and their variants have been used to detect the closest SNBP in [14–17]. The analysis of voltage stability of power systems and computation of maximum loadability while incorporating dynamic constraints along with steady-state constraints has been explored in [18–24]. Wide-area-measurement-based voltage stability analysis using modified coupled single-port models has been examined in [25]; while physical constraints such as var limits are not considered in that work, they are shown to be important.

Ultimately, the power-flow (PF) equations and their solution is at the core of voltage stability analyses and the aforementioned voltage stability analysis methods use iterative solution methods which suffer from non-convergence issues. Two classes of iterative methods for solving the PF problem are well known, Gauss–Seidel and Newton Raphson (NR) (notable is the Fast Decoupled Load Flow) [26–30], both of which have many variants developed to improve the convergence properties ([31–34] to cite only a few.) The problems with these classes of solvers are also well known: initial estimate dependence [27,28]; non-convergence when a solution exists and convergence to the wrong (low-voltage/large angle) solution [35]. To date, no variants within these classes have been shown to deal with these problems in a consistent way. Often, the convergence issues are exacerbated near the bifurcation point. The most popular class of industrial methods, the NR class,

* Corresponding author.
E-mail addresses: srao12@asu.edu (S.D. Rao), tylavsky@asu.edu (D.J. Tylavsky), yang-feng@siemens.com (Y. Feng).

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converges reliably for systems with nominal voltage profiles [36], but often struggles near the SNBP.

The method used in most PF applications to estimate the SNBP is the continuation power flow (CPF), which is an NR-based method [37]. The computational complexity of the CPF is much higher than a simple NR PF since it requires solving a new PF problem at each step as one moves along the P–V curve toward the SNBP [37]. The problems of a singular Jacobian matrix occurring at the SNBP are eliminated in [38] by reformulating the power-flow problem and introducing a new bus type called AQ bus which has the voltage angle and reactive power consumption specified.

The holomorphic embedding method (HEM) is a more recently developed class of nonlinear equation solvers that is guaranteed to converge to the operable solution for the basic PF problem, if it exists, provided the conditions of Stahl's theorem are satisfied [39,40,42]. It is recursive, rather than iterative, and can be configured so that the initial state needed to guarantee the above properties can be easily calculated. The advantages of the HEM can be exploited to develop methods that can reliably estimate the SNBP of a system.

The HEM uses the concept of holomorphic embedding (HE) to convert the non-holomorphic power-balance equations (PBE's) of the PF problem into a set of holomorphic functions. A holomorphic function is a complex-valued analytic function, which has the property that it is infinitely complex differentiable around every point within its domain. One property of holomorphic functions important for the purpose here is that they can be represented by their Taylor series in a neighborhood of each point in their domain [47].

Using the HE formulation, the voltages at all buses and the reactive power at the PV buses are expressed as Maclaurin series of a complex embedding parameter, α. With the properties of holomorphic functions and the use of analytic continuation, Stahl's theorem [40] guarantees that, if an operable solution exists at the given loading level, the correct voltage solution will be obtained using Padé approximants of the holomorphic series as long as the correct germ is used [39,43]. (The germ for the HEM is analogous to the initial estimate of the solution in the NR method and will be explained in later sections. However, unlike the NR initial estimate, the germ must be systematically obtained by solving a set of equations.)

The methods in the literature that calculate the SNBP rely on solving successive problems, each of which has the complexity of the PF problem. This paper presents four HEM-based methods that estimate the SNBP of a system, three of which do not require multiple PF problems to be solved. All of these methods rely on an important property of a Padé approximant, that it is the maximal analytic continuation of the given function [40].

The paper is organized as follows: Discussed in Section "Two HEM formulations" is a formulation that allows load and real-power-generation extrapolation (a property critical to the three approaches proposed here) for the power-flow problem. Also discussed in this section is a formulation previously published and why it cannot be used for load extrapolation. Section "Padé approximants and branch cuts" contains the method of using the roots of Padé approximants to find the SNBP of the system and the fundamental theory behind this approach. In Section "The sigma methods", two so-called "sigma methods" used to identify weak nodes of the power system and to estimate the SNBP of the system are introduced [44]. In Section "Numerical results for scaling all loads uniformly", the results of numerical experiments that compare the SNBP's predicted by different methods when all loads are scaled uniformly are presented. Section "Formulation to allow loads at different buses to be scaled by different amounts" contains the development of a formulation that can be used to scale the load vector in an arbitrary direction and, consequently, be used to obtain the SNBP. Section "Numerical results with loads at different buses scaled by different amounts" provides the numerical results for the formulation described in Section "Formulation to allow loads at different buses to be scaled by different amounts". Section "Incorporating var limits in the SNBP estimation" presents the results when var limits are accounted for and Section "Proposed ZIP load model for the HEM" provides the formulation for polynomial ZIP load models. Finally, conclusions are presented in Section "Conclusion".

Two HEM formulations

To apply the HEM to a complex-valued problem requires that the system of equations to be solved be holomorphic. Because of the presence of the complex conjugate operator, the traditional PF equations are not holomorphic. Hence the first step in developing a proper HEM formulation is to render the PBE's holomorphic.

Consider a general (N)-bus system consisting of a slack bus, called *slack*, a set m consisting of PQ buses, a set p consisting of PV buses which are not on var limits and a set q consisting of PV buses on maximum/minimum var limits. The PBE for a PQ bus with a constant power load is given by

$$\sum_{k=1}^{N} Y_{ik} V_k = \frac{S_i^*}{V_i^*}, \quad i \in m \tag{1}$$

where, Y_{ik} is the (i,k) element of the bus admittance matrix, and S_i, and V_i are the complex power injection and voltage at bus i, respectively. (The HEM model for polynomial ZIP loads will be discussed in Section "Proposed ZIP load model for the HEM".)

The traditional defining equations for a PV bus are given by (2) and (3).

$$P_i = Re\left(V_i \sum_{k=1}^{N} Y_{ik}^* V_k^*\right), \quad \forall i \in p \tag{2}$$

$$|V_i| = V_i^{sp}, \quad \forall i \in p \tag{3}$$

where P_i denotes the real power injection and V_i^{sp} is the specified voltage magnitude at bus i. PV buses on var limits are treated similar to PQ buses with their reactive power generation fixed at the appropriate limit and the real power generation given by (2).

Formulation that allows extrapolation of the load

The above non-holomorphic equations can be holomorphically embedded in an infinite number of ways. It is possible to embed (1)–(3) in such a way that the solution obtained at different values of real α, represents the solution (if it exists) when the complex power injections at the load buses and real power at generation buses are scaled by a factor of α. It is necessary to have such a formulation in order to be able to estimate the SNBP of the system without having to solve a new PF problem at different loading conditions. The HEM formulations published in the past, do not allow one to scale the load by a factor of α, since they solve the given power-flow problem at α = 1.0, because they are consistent with the power system equations only at α = 1.0. This will be explained in more detail in Section "Formulation with a simple germ". Consider the following set of holomorphically embedded equations, where (4) represents the PBE for PQ buses, (5) represents the voltage magnitude constraint for the slack bus, (6) represents the PBE for the PV buses, (7) represents the voltage magnitude constraint for the PV buses and (8) represents the PBE for PV buses on var limits.

$$\sum_{k=1}^{N} Y_{ik} V_k(\alpha) = \frac{\alpha S_i^*}{V_i^*(\alpha^*)}, \quad i \in m \tag{4}$$