Electrical Power and Energy Systems 84 (2017) 55–63

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

A concise, approximate representation of a collection of loads described by polytopes ☆

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ARTICLE INFO

Article history:
Received 28 November 2015
Received in revised form 8 March 2016
Accepted 3 May 2016

Keywords:
Demand response
Load aggregation
Minkowski sum
Polytope
Linear programming

ABSTRACT

Aggregations of flexible loads can provide several power system services through demand response programs, for example load shifting and curtailment. The capabilities of demand response should therefore be represented in system operators' planning and operational routines. However, incorporating models of every load in an aggregation into these routines could compromise their tractability by adding exorbitant numbers of new variables and constraints.

In this paper, we propose a novel approximation for concisely representing the capabilities of a heterogeneous aggregation of flexible loads. We assume that each load is mathematically described by a convex polytope, i.e., a set of linear constraints, a class which includes deferrable loads, thermostatically controlled loads, and generic energy storage. The set-wise sum of the loads is the Minkowski sum, which is in general computationally intractable. Our representation is an outer approximation of the Minkowski sum. The new approximation is easily computable and only uses one variable per time period corresponding to the aggregation's net power usage. Theoretical and numerical results indicate that the approximation is accurate for broad classes of loads.

Introduction

Demand response (DR), the coordinated control of flexible loads, can render great benefits to power systems and is recognized as an essential new source of flexibility for renewable integration [1]. DR activities are now widely engaged in by third party companies, utilities, and system operators. Comprehensive surveys on DR are provided by [2–4]. In this paper, we refer to the entity controlling a collection of loads as the load aggregator.

System operators must integrate DR into their operational routines to fully leverage its capabilities. For example, multiperiod optimal power flow or unit commitment can be used to perform load-shifting using DR alongside energy storage [5,6]. This is challenging because the loads in DR programs are often small, diverse, and numerous; a typical aggregation may contain upwards of 10^6 loads. Exactly representing the loads of multiple DR aggregations within multiperiod optimal power flow could add millions of new variables and constraints, making it computationally intractable [7–9]. Moreover, the individual load models may be known to the load aggregator but not the system operator.

To overcome these difficulties, load aggregators need concise models of their loads' aggregate characteristics, thus enabling them to share their capabilities with the system operator without describing every load individually. System operators can then straightforwardly incorporate such a model into tasks like multiperiod optimal power flow or unit commitment as they would a conventional resource like grid-scale storage [10]. Because the model is concise, i.e., consisting of a small number of variables and constraints, it does not increase the difficulty of the system operator's tasks. We further discuss the role of DR and concise modeling within multiperiod optimal power flow in Section "Role within power system operations".

In this paper, we develop a concise, approximate representation for aggregations of loads modeled by convex polytopes, i.e., sets of linear constraints. Since we only deal with convex polytopes, we will henceforth omit the term 'convex' and simply write 'polytope'. The set-wise sum of two sets is called the Minkowski sum, and is computationally intractable even for polytopes. As observed in [8,11], the flexibility of an aggregation of polytopic loads is captured by the Minkowski sum, which we define in Section "Load aggregation as Minkowski sums". Approximate Minkowski sums are an active research area, but most work focuses on the

☆ This work is funded by The Natural Sciences and Engineering Research Council of Canada.

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http://dx.doi.org/10.1016/j.ijepes.2016.05.001
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calculation of two and three-dimensional sums of highly complex polytopes as in [12,13]. In Section "Extension to general polytopes", we develop a novel outer approximation of the Minkowski sum, which is easily computable in polynomial-time. Our method is generally applicable regardless of dimension, and also results in a polytope in $\mathbb{R}^D$, where D is the number of time periods. This makes it easy to incorporate into optimization routines for power system operations without sacrificing tractability.

A number of existing papers describe techniques for concisely modeling large collections of loads, which we now summarize. Work on this topic has been on-going since the 1980s beginning with [14] and more recently in [15–17], which model the probability distribution of temperatures in spaces controlled by thermostatic loads using a partial differential equation. Thermostatic loads are a particular focus area within DR work as they represent almost 20% of load in industrialized countries such as the U.S. [18]. In [19], the authors model the control of a collection of thermostatic loads using a second-order LTI system and design a controller to achieve desired power outputs and then return the aggregate system to steady-state.

Our work is closely related to several recent papers that approximate a collection of loads as generalized energy storage. In [9], charging electric vehicles are modeled as deferrable loads, and analytical generalized storage expressions for their aggregate capabilities are obtained. Aggregations of thermostatic loads are approximated as time-varying thermal batteries in [20,21] and as generalized batteries in [8]; the latter derives inner and outer generalized battery models to represent a collection of thermostatic loads. The storage models obtained in these papers consist of linear constraints, similar to the polytope-based framework employed in this paper. In [22], the exact Minkowski sum is identified as a measure of a power systems total flexibility. In [23,11,24], the Minkowski sum is identified as the aggregate flexibility of a collection of loads, and is used to quantify and visualize flexibility. In [11], reduced-order stochastic-hybrid models are developed for multiple types of loads, which can then be combined. Their approach approximates the Minkowski sum using a discrete approximation of load parameters and states, which allows them to incorporate uncertainty such as random electric vehicle arrivals. Whereas our approach is most suitable under steady-state conditions, their approach is designed for real-time control.

The contributions of the paper are as follows:

- An novel outer approximation is given for the Minkowski sum of flexible loads described by polytopes with the same A matrix (shape) in Section "Polytopes with the same shape". Note that we make no approximation to the individual loads' parameters beyond the polytope assumption.
- We extend the outer approximation to the Minkowski sum of general polytopes in Section "Extension to general polytopes", and give a procedure for optimizing the approximation.
- We numerically characterize the performance of the approximation in Section "Numerical examples" using volume-based error metrics and by simulation within optimal power flow.
- We show analytically that the approximation is exact for loads described by hypercubes and simplices Section "Analytical results".

Background

Notation

A polytope is a set in $\mathbb{R}^D$ whose boundary is composed of flat surfaces called facets [25]. These facets are derived from hyperplanes and are sets in $\mathbb{R}^{D-1}$. We denote polytopes using bold script with subscripts for differentiation between them, e.g., $\mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_k$. We restrict our attention to polytopes that are closed and bounded, i.e., compact.

The points within a polytope can be represented as convex combinations of the extreme points of the polytope [26]. We denote points (or vectors) using lowercase italicized letters, e.g., x, y. The set of vertices of such polytopes then form a minimal unique (up to ordering) representation for a polytope. Such a representation is referred to as the V-representation of a polytope. Sets of vertices are denoted using uppercase letters with a bar, e.g., $\overline{X}, \overline{Y}$.

An alternate representation for a polytope is as the intersection of a collection of half-spaces (referred to as the H-representation of a polytope). In the H-representation, each half-space generates a facet of the polytope and is represented as a linear inequality, e.g., $a^T x \leqslant b$. A minimal H-representation contains only inequalities corresponding to facets of the polytope with non-zero area, and is unique up to ordering and scaling. The H-representation is generally preferred to the V-representation for DR because it is the form of almost all load models.

We use uppercase letters to represent matrices and subscripts to indicate that a set or matrix is associated with a particular polytope. We may write the H-representation of a polytope in matrix form as $A_1 x \leqslant b_1$, and denote it by the matrix–vector pair (A_1, b_1). The polytope can also be written explicitly as $\mathbf{P}_1 = \{x \,|\, A_1 x \leqslant b_1\}$ We use the term *A-matrix* to refer to the matrix A_1 of a polytope in H-representation.

Example 1. Consider a triangle in $\mathbb{R}^2$. In, V-representation, we may denote it by its set of vertices as $\overline{X}_1 = \{(0,0),(1,0),(0,1)\}$. In H-representation, we may denote it by the matrix–vector pair (A_1, b_1), where

$$A_1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Note how in this case, the vertices of the polytope are generated by solving the equalities associated with each inequality. In general, the vertices of a polytope will be generated from the solution of equalities associated with adjacent facets.

V-representations and H-representations of a polytope can be derived from each other. Conversion from the H-representation to the V-representation is known as vertex enumeration; the reverse problem is known as facet enumeration. Unfortunately both of the above problems are, in general, NP-hard [27]. For polytopes that are bounded, the complexity of vertex and facet enumeration remains open [28]. No tractable solutions to these problems are currently known.

Additionally, while the above refers to minimal V-representations and H-representations, both may contain redundant information. In the V-representation, this implies the inclusion of points lying inside the polytope. In the H-representation, this implies the inclusion of non-binding inequalities (i.e. inequalities that do not generate a facet of the polytope as their associated hyperplanes either lie outside the polytope or are tangent to it at a single point). Testing a component of either representation for redundancy can be done with linear programming [29].

Role within power system operations

Large aggregations of flexible loads are valuable resources for power system operators and hence should be represented in power system dispatch routines. Multi-period optimal power flow is a standard approach to dispatching power systems with dynamic constraints such as ramping and storage capacity limits [30]. Since