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A singular value decomposition-based technique for decoupling and analyzing power networks

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ABSTRACT

An eigen-based transformation called Channel Components Transform (CCT) has been recently proposed for decoupling power networks. CCT has revealed many unique features in power system studies especially voltage stability analysis and monitoring. However, this method may face some difficulties in certain situations. The difficulties including the non-existence difficulty and the robustness difficulty are discussed in this paper. This paper will then show that by using the singular value decomposition instead of the eigen-decomposition, the difficulties can be effectively overcome. Therefore, a new transformation based on singular value decomposition is proposed. An algorithm is also presented which shows how the proposed transformation can be applied to voltage stability analysis. The proposed algorithm is verified by using several standard systems as case studies.

Introduction

Many reasons such as economic and environmental constraints make it difficult to add new facilities such as transmission lines to power systems. On the other hand, the electrical power demand has increased significantly over the last years. Consequently, utilities are forced to operate their systems close to the stability limits. In this situation, voltage stability is a major concern since a disturbance in the system can simply lead to a voltage collapse. This risky situation has led to a great deal of research works performed by many researchers, and as a result, many methods have been proposed for voltage stability analysis, monitoring, and improvement [1–8].

One of the most recent methods called the Channel Components Transform (CCT) was proposed in [9]. CCT is a transformation technique which is based on eigen-decomposition of the impedance matrix. CCT decouples a complex power system into a set of single-branch networks called channel circuits. Ref. [9] has shown that among all channels of a system, one channel (critical channel) is most responsible for voltage collapse. Critical channel can reveal the critical modes of a voltage collapse. Using the CCT, [9] has proposed a framework for voltage stability analysis. Using the proposed framework, different aspects of voltage stability can be investigated. For example, the buses which are weak with respect to voltage stability can be determined [9]. Similarly the most critical generators, and the weakest transmission lines can be identified [10]. Ref. [10] also showed that CCT can be utilized for online voltage stability monitoring using a limited number of Phasor Measurement Units (PMU). Ref. [12] extended the application of CCT to design shunt reactive support to improve voltage stability.

In spite of many unique features that CCT offers, there are two difficulties since it is based on eigen-decomposition:

- A theoretical difficulty is that the decomposition does not always exist. In other words, there might be an impedance matrix which could not be diagonalized using eigen-decomposition.
- A numerical difficulty is that, even if the decomposition exists, it might not provide a basis for robust computation. That means if some errors exist in the data (such as in the impedance matrix), these errors could be magnified in the transformed variables. Therefore, we might not be able to get an acceptable accuracy in the results.

This paper aims to overcome these difficulties. The difficulties are discussed in detail first. The paper will then show that by using Singular Value Decomposition (SVD) instead of the eigenvalue decomposition, the problems can be solved. Therefore, a decoupling transformation technique which is based on SVD is proposed in this paper. An algorithm for voltage stability analysis using the proposed transformation is also presented. The performance of the algorithm is then verified using several standard case studies.

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Limitations of eigen-decomposition

The eigen-decomposition has two difficulties: (A) non-existence difficulty, and (B) Robustness difficulty. In this section, these limitations are discussed in detail.

Non-existence difficulty

Assume that matrix [Z] is to be decomposed using eigen-decomposition. If [Z] is diagonalizable, the matrix has an eigenspace decomposition. If the matrix is not diagonalizable, then it is called defective, and, it cannot be decomposed into eigenspaces. In particular, an $n \times n$ matrix is defective if and only if it does not have n linearly independent eigenvectors.

From linear algebra, we know that if an $n \times n$ complex matrix is Hermitian, then there exist n linearly independent eigenvectors for this matrix. In other words, any Hermitian matrix can be diagonalized using eigen-decomposition. If a complex matrix is not Hermitian, there would be no guarantee that it can be diagonalized. Matrix [Z] is Hermitian if and only if it satisfies the following conditions

$$Z = Z^H \tag{1}$$

where H stands for complex conjugate transpose.

A power system impedance matrix [Z] may be symmetric i.e. $Z = Z^T$ (where T stands for transpose) but definitely it will not be Hermitian i.e. $Z \neq Z^H$. Since the impedance matrix is not Hermitian, there is no guarantee that we are able to decompose it into a diagonal matrix using eigen-decomposition.

Robustness difficulty

The eigenvalues of some matrices could be sensitive to perturbations. In other words, small changes in the matrix elements could lead to large changes in the eigenvalues. Assume that [Z] has a full set of linearly independent eigenvectors and it can be decomposed using eigenvalue decomposition as follows:

$$Z = T^{-1} \Lambda T \tag{2}$$

$$\text{or } \Lambda = TZT^{-1} \tag{3}$$

Now let δZ denote some change in Z. Then,

$$\Lambda + \delta\Lambda = T(Z + \delta Z)T^{-1} \tag{4}$$

$$\text{Hence } \delta\Lambda = T(\delta Z)T^{-1} \tag{5}$$

Taking matrix norms,

$$\|\delta\Lambda\| \leqslant \|T\| \|T^{-1}\| \|\delta Z\| = k(T)\|\delta Z\| \tag{6}$$

where $k(T)$ is the condition number of T, the matrix of eigenvectors. The above analysis implies that a perturbation in Z can be magnified by a factor as large as $k(T)$.

As discussed in [9], the Z matrix is obtained from SCADA data. So one would expect some errors in the Z matrix. This kind of errors is an example for perturbation in Z. Similarly, roundoff errors introduced during the computation of eigenvalues have the same effect as perturbations in the original matrix [11]. Consequently, these errors may be magnified in the computed eigenvalues. If the transformation matrix has a large condition number, the errors would be significantly magnified, leading to unreliable results.

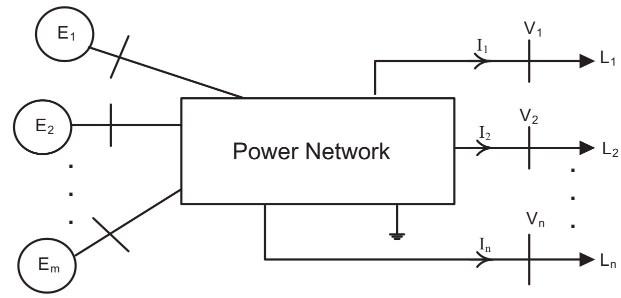

Fig. 1. A general electric power network.

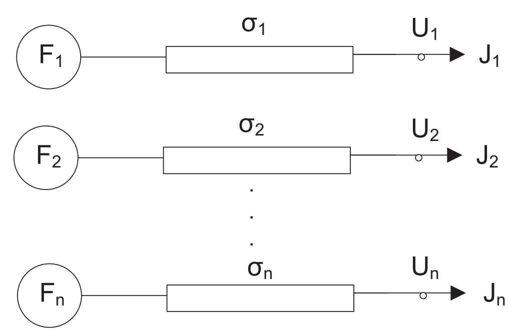

Fig. 2. Channel domain representation of a complex network.

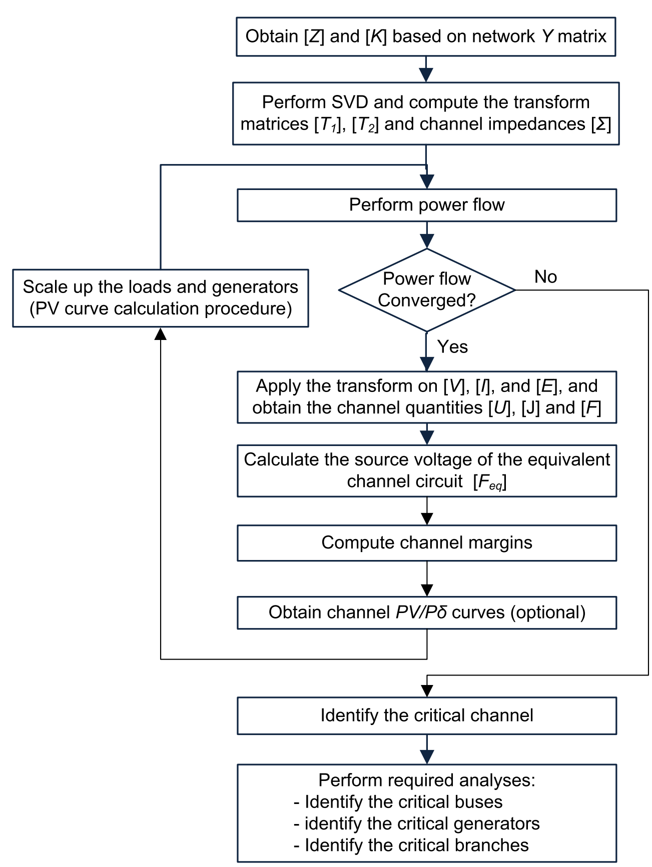

Fig. 3. Procedure of the voltage stability analysis.