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Volt/var optimization of unbalanced distribution feeders via mixed integer linear programming

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ABSTRACT

The paper presents a Mixed Integer Linear Programming (MILP) model for the solution of the three-phase volt/var optimization (VVO) of medium voltage unbalanced distribution feeders. The VVO of a distribution feeder is aimed at calculating the most efficient operating conditions by means of the scheduling of transformers equipped with an on-load tap changer and distributed reactive power resources (such as embedded generators and switchable capacitors banks). The proposed model allows the representation of feeders composed by three-phase, two-phase, and single-phase lines, by transformers with different winding connections, by unbalanced wye- and delta-connected loads, by three-phase and single phase capacitor banks and embedded generators. The accuracy of the results is verified by using IEEE test feeders.

Introduction

One of the active network management functions that modern distribution management systems are expected to provide is the so-called volt/var optimization (VVO) [1,2].

Different VVO definitions have been used in the technical literature. In general VVO problem refers to the determination of the set points of on-load tap changers (OLTCs) and switchable capacitor banks regulators with the objective of loss minimization or load reduction, taking into account the usual operating constraints such as minimum and maximum voltage limits and ampacity (e.g. [3]). Modern and future distribution networks also require the optimization of the reactive power compensation capabilities provided by embedded generators (EGs) and by storage systems, with particular reference to those equipped by power electronic converters such as photovoltaic inverters (e.g. [4,5] and references therein).

The typical differences between the VVO in distribution networks and the optimal power flow (OPF) problem in transmission systems are:

- usually VVO does not involve generator active power outputs as control variables, since EG outputs are mainly determined by the availability of energy resources and market/regulatory constraints;
- the main VVO control variables are integer (transformer taps and capacitor switching), whilst typical OPF control variables are continuous;
- voltage limits are usually more stringent than ampacity constraints;
- line and load unbalances appear in general more significant in distribution feeders than in transmission networks (this justifies the modelling effort in order to represent all the three phases of the system [6]).

Moreover, with respect to transmission system lines, distribution feeders are characterized by shorter length, higher ratio between resistance and reactance of the longitudinal impedance, and they are expected to transmit lower power flows. Therefore the influence of active power flows to voltage profile is not negligible and, usually, the maximum phase difference between the voltages at a bus in different operating conditions is limited to few degrees. On the other hand VVO shares several aspects and solution approaches with OPF, so to justify the definition of distribution optimal power flow (DOPF) models (e.g. [7]).

This paper aims at describing a Mixed Integer Linear Programming (MILP) model for the solution of the deterministic VVO of unbalanced distribution feeders. As reviewed in [8], among the various approaches adopted in the literature, the use of Mixed Integer Linear Programming (MILP) appears to have been less explored than other methods for the solution of VVO problems. The motivation of using a MILP approach for VVO lays mainly on the availability of efficient solvers for this class of problems [9].

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This approach has been recently investigated not only for distribution networks (e.g. [10–13]) but also for the solution of the classical OPF problem in transmission systems (e.g. [14,15]).

The proposed model is a development of the one presented in [8] based on the representation of the distribution network with voltage – current relationships in Cartesian coordinates. The linear representation of loads and EGs is based on the assumption of limited deviations of bus voltage phasors with respect to the corresponding rated phasors. The MILP formulation presented in [8] assumes that the network is balanced. This paper presents a MILP model for a generic unbalanced distribution feeder, taking into account the different types of transformer winding connections, wye- and delta-connected loads, the presence of single-phase and three-phase EGs as well as the possibility to switch the capacitors on a per phase basis. The unbalanced per phase model is based on the representation of both lines and transformers with an equivalent network of uncoupled branches [16].

As in [8] the configuration of the feeder is assumed known and it is not changed. The developed MILP model determines the optimal solution only for the current time. Time coupling constraints, with particular reference to the maximum number of transformers tap changes and on/off switching cycles of capacitor banks, are not included. They would require an optimal schedule for the entire day (e.g., [2,7]). If the optimization horizon is larger than some tens of minutes, several uncertainties relevant to both renewable generators outputs and loads levels need to be taken into account (see, for example, [17]).

As expected, the required computational effort is significantly increased with respect to the equivalent single phase model presented in [8].

The structure of the paper is the following. Section 'Description of the optimization problem' presents the main characteristics of the implemented MILP model and the complete model formalization. Section 'Test results' illustrates the performances by the results obtained for some IEEE test feeders [18] with additional EGs.

Description of the optimization problem

This section presents the implemented formulation of the MILP model. The network has $N/3$ three-phase buses each composed by 3 nodes plus a slack bus (that represents the HV bus of the HV/MV substation) also composed by 3 nodes. The network has N_b uncoupled branches and it is composed by the equivalent circuits of lines and transformers described in one of the following subsections. Voltages of each node and currents of each branch are represented by the Cartesian coordinates of their phasors: (V^{re}, V^{im}) and (I^{re}, I^{im}), respectively. Voltage phasors at the nodes of the slack bus have modulus equal to the rated value and phases so that to form a three-phase symmetrical system. All the equations are written in per unit (pu).

Objective function

As in [8], the considered deterministic static VVO aims at minimizing the active power consumption from the transmission network (and maximizing the active power injection), whilst minimizing the violation of the minimum and maximum voltage limits at all the buses and the violation of a minimum power factor constraint enforced for each phase at the slack bus:

$$\min\left\{\sum_{k_s=1}^{3}P_{k_s}+w_{\min}\sum_{k=1}^{N}\chi_k^{\min}+w_{\max}\sum_{k=1}^{N}\chi_k^{\max}+w_{\rm pf}\sum_{k_s=1}^{3}\chi_{k_s}^{\rm pf}\right\} \tag{1}$$

where P_{k_s} is the active power absorbed from the HV external network through node k_s of the slack bus, $\chi_k^{\min}$ and $\chi_k^{\max}$ indicate the violations of the minimum and maximum voltage bounds at node k, respectively, $w_{\min}$ and $w_{\max}$ are the weights that penalize the corresponding violations, $\chi_{k_s}^{\rm pf}$ indicates the violation of the minimum power factor at node k_s of the slack bus and $w_{\rm pf}$ the relevant penalizing weight.

For each node k, $\chi_k^{\min}$ and $\chi_k^{\max}$ are provided by a piecewise linear function (PLF) of the square of the voltage violation. The set of constraints that define the PLF are the following

$$\begin{aligned}-\chi_k^{\min}+\lambda_i^{\Delta V}\Delta V_k^{\min}\leqslant-\Lambda_i^{\Delta V}\quad\text{and}\quad-\chi_k^{\max}+\lambda_i^{\Delta V}\Delta V_k^{\max}\\ \leqslant-\Lambda_i^{\Delta V}\quad\forall i=1\ldots\bar{z}_{\Delta V}-1\end{aligned} \tag{2}$$

where $\Delta V_k^{\min}$ and $\Delta V_k^{\max}$ are the non-negative continuous variables that correspond to the absolute value of the voltage violations of the minimum or maximum limits at node k (i.e., $V_k^{\min}$ and $V_k^{\max}$), respectively; $\lambda_i^{\Delta V}$ is the slope and $\Lambda_i^{\Delta V}$ is the ordinate intercept of PLF interval i; $\bar{z}_{\Delta V}$ is the number of PLF breakpoints.

Slope $\lambda_i^{\Delta V}$ and ordinate intercept $\Lambda_i^{\Delta V}$ are calculated as:

$$\lambda_i^{\Delta V}=\Delta V_{i+1}+\Delta V_i,\qquad\Lambda_i^{\Delta V}=\Delta V_i^2-\lambda_i^{\Delta V}\Delta V_i\quad\forall i=1\ldots\bar{z}_{\Delta V}-1. \tag{3}$$

For node k_s corresponding to one the three phases of the slack bus, $\chi_{k_s}^{\rm pf}$ is obtained by the following inequality constraint

$$|Q_{k_s}|-\tan(\cos^{-1}(pf_{\min}))|P_{k_s}|-\chi_{k_s}^{\rm pf}\leqslant0 \tag{4}$$

where Q_{k_s} is the reactive power abortion from the HV external network through node k_s of the slack bus and $pf_{\min}$ is the power factor minimum operating value. P_{k_s} and Q_{k_s} value are linked with the currents in the branches connected to node k_s by the following equality constraints

$$P_{k_s}-\sum_{b_s\in B_s}V_{k_s}^{re}I_{b_s}^{re}-\sum_{b_s\in B_s}V_{k_s}^{im}I_{b_s}^{im}=0 \tag{5}$$

$$Q_{k_s}+\sum_{b_s\in B_s}V_{k_s}^{re}I_{b_s}^{im}-\sum_{b\in B_s}V_{k_s}^{im}I_{b_s}^{re}=0 \tag{6}$$

where V_{k_s} is the fixed voltage phasor at node k_s, B_s is the set of branches b_s connected to node k_s of the slack bus and I_{b_s} is the current in branch b_s. Absolute values $|P_{k_s}|$ and $|Q_{k_s}|$ are obtained by

$$\begin{aligned}&|P_{k_s}|-\Delta P_{k_s}^{+}-P_{k_s}=0\quad\text{and}\quad|Q_{k_s}|-\Delta Q_{k_s}^{+}-Q_{k_s}=0\\&\Delta P_{k_s}^{+}+M_{k_s}^{P}w_{k_s}^{P}\leqslant M_{k_s}^{P}\quad\text{and}\quad\Delta Q_{k_s}^{+}+M_{k_s}^{Q}w_{k_s}^{Q}\leqslant M_{k_s}^{Q}\\&-|P_{k_s}|+\Delta P_{k_s}^{-}-P_{k_s}=0\quad\text{and}\quad-|Q_{k_s}|+\Delta Q_{k_s}^{-}-Q_{k_s}=0\\&\Delta P_{k_s}^{-}-M_{k_s}^{P}w_{k_s}^{P}\leqslant0\quad\text{and}\quad\Delta Q_{k_s}^{-}-M_{k_s}^{Q}w_{k_s}^{Q}\leqslant0\end{aligned} \tag{7}$$

where $\Delta P_{k_s}^{-}$, $\Delta P_{k_s}^{+}$, $\Delta Q_{k_s}^{-}$, $\Delta Q_{k_s}^{+}$ are non-negative continuous variables, $w_{k_s}^{P}$ and $w_{k_s}^{Q}$ are binary variables, and $M_{k_s}^{P}$ and $M_{k_s}^{Q}$ are tight big values.

As in [8], other than those shown in (1), the objective function includes additional terms that allow the penalized disconnection of loads and EGs by means of binary variables if it is needed to find a feasible solution.

Branch equations

The equivalent representation as a circuit of uncoupled branches for both lines and transformers (with various types of winding connections) have been presented in several papers (e.g. [16,19–21]). Here below we review the models used in this paper to obtain the test results.

The three-phase line data are the impedance and susceptance 3×3 symmetrical matrices (**Z** and **B**). Analogously to three-phase lines, the input data for two-phase and single-phase lines are represented by 2×2 matrices and two complex numbers,