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Optimal distribution systems reconfiguration for radial and meshed grids ☆

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ABSTRACT

In recent years, convex relaxations of the AC power flow equations have gained popularity. Their success is mainly due to the optimality guarantees they can provide. However, their use has mostly been limited to continuous problems. This paper extends the reach of relaxations to reconfiguration problems with binary decision variables, such as minimal power loss, load balancing and power supply restoration. This is achieved by extending the relaxations of AC power flow equations to bear on the on/off nature of constraints featured in reconfiguration problems. This leads to an approach producing AC feasible solutions with provable optimality gaps, on both radial and meshed grids. In terms of run time, the new models are competitive with state-of-the-art approximations which lack formal guarantees.

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Introduction

The steady-state alternating current (AC) power flow equations are at the core of every computational problem in power systems. These equations form a system of non-convex constraints, raising significant challenges in optimization frameworks. Global nonlinear programming (NLP) solvers are unable to scale on small size instances. Problems including binary decision variables, e.g. reconfiguration and power supply restoration, are even more challenging. Yet, utilities are facing an increasing need for efficient reconfiguration tools optimizing grid operations and planning. This is especially relevant with the increasing penetration of distributed renewable generation and its intermittent nature.

There is a rich set of optimization approaches trying to tackle problems in power systems. Black-box heuristic methods, which push power flow calculations outside the optimization solver, are widely used, owing to their broad applicability [1–3]. Yet, these approaches fail to exploit the problem structure and lack formal guarantees on the quality of the solutions returned. More recently, linear and quadratic approximations of power flow equations attracted a number of studies [4–7]. This enables the use of a new generation of mathematical programming solvers including mixed-integer linear (MIP), quadratic (QP) and second-order cone (SOCP) programming solvers [8]. While approximations also suffer from a lack of optimality guarantees, the solution returned may be infeasible with respect to the original AC model. This is particularly true with reconfiguration problems, as switching moves us away from the nominal configuration under which the approximations are valid [9].

Therefore, recent research has turned to finding linear or convex *relaxations* of the AC power flow equations [10–13] and constructing heuristics based on such relaxations [14]. These convex relaxations are of great interest since they efficiently produce lower bounds on the quality of the feasible solutions generated by global or local optimization techniques. For purely continuous problems such as optimal power flow (OPF), the semi-definite programming (SDP) relaxation in [12] is often tight, i.e. its solution is also a solution to the original problem. However, for reconfiguration and power supply restoration problems, the applicability of such relaxations has not been investigated; the only exception we are aware of is the work of Jabr et al. [15] discussed below. Such an investigation is the topic of the present paper.

Our contributions are as follows. We extend recent quadratic relaxations of power flow equations for radial [11] and meshed networks [13] to include topology changes and bear on reconfiguration problems. Key to obtaining effective relaxations is an adequate representation of on/off constraints, which we describe in some detail. Studying both radial and non-radial topologies

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enables us to efficiently exploit the current radial topology of most distribution systems, whilst also accounting for more general topologies representative of advanced networks and microgrids with higher penetration of distributed generation. We then use these relaxations to obtain lower bounds and solve reconfiguration problems with objectives such as loss minimization and load balancing, as in [7], as well as power supply restoration problems.

Qualitatively, our approach leads to AC feasible solutions with provable optimality gaps, of zero in some cases. Except for the work of Jabr et al. [15], we are not aware of other approaches capable of providing such guarantees for reconfiguration problems. Jabr et al. use a different relaxation to solve loss minimization problems, exclusively with radial topologies. In the meshed case, and for power supply restoration (regardless of topology), we are not aware of any report of provable quality gaps. Quantitatively, for radial topologies, our approach leads to better computation times when compared to existing approaches based on Tabu search [16], mixed-integer second-order cone approximations [7], and relaxations [15]. Our experimental results also suggest that meshed topologies are often beneficial with reductions in the objective function up to 2–15%.

The paper is organized as follows. Section 'Power flow equations and their relaxation' reviews the power flow equations suited to the radial and meshed case, as well as their relaxation given in [11,13], respectively. Section 'Relaxations for reconfiguration problems' extends these relaxations to account for the specificities of reconfiguration problems and describes our overall approach. Section 'Applications and experimental results' describes the three specific reconfiguration problems we consider, and presents an experimental evaluation of our approach on these problems.

Power flow equations and their relaxation

In the following, we consider a network $\langle N, E\rangle$, where N is the set of buses (nodes), and E is the set of directed lines. Note that if two nodes i and j are connected, E contains both (i,j) and (j,i). E^r will denote the set of lines restricted to one direction, i.e. $i < j$.

The relevant variables at the node level are voltage magnitudes v_i, and phase angles θ_i. On the line level (i,j) we have active power p_{ij} and reactive power q_{ij}. Line properties are either represented via resistance $\boldsymbol{r}_{ij}$ and reactance $\boldsymbol{x}_{ij}$ or via susceptance $\boldsymbol{b}_{ij}$ and conductance $\boldsymbol{g}_{ij}$.

We assume the following set of operational constraints, where $\boldsymbol{S}^u_{ij}$ is the thermal limit (capacity) on line $(i,j) \in E$, $\boldsymbol{\theta}^u$ is the maximal phase angle difference between connected buses, and $(\boldsymbol{v}^l, \boldsymbol{v}^u)$ represent the lower and upper bounds on voltage magnitudes.

$$p_{ij}^2 + q_{ij}^2 \leqslant \boldsymbol{S}^u_{ij}, \quad (i,j) \in E \tag{1a}$$

$$-\boldsymbol{\theta}^u \leqslant \theta_i - \theta_j \leqslant \boldsymbol{\theta}^u, \quad (i,j) \in E \tag{1b}$$

$$\boldsymbol{v}^l \leqslant v_i \leqslant \boldsymbol{v}^u, \quad i \in N. \tag{1c}$$

Radial distribution networks

Distribution systems have a meshed structure, but are often operated radially. From an optimization point of view, the radial topology makes it possible to exploit the quadratic *DistFlow* equations [1], resulting in more efficient computation. These equations are defined as

$$p_{ij} = \sum_{k:(j,k)\in E} p_{jk} + \boldsymbol{r}_{ij}\frac{p_{ij}^2 + q_{ij}^2}{v_i^2} \tag{2a}$$

$$q_{ij} = \sum_{k:(j,k)\in E} q_{jk} + \boldsymbol{x}_{ij}\frac{p_{ij}^2 + q_{ij}^2}{v_i^2} \tag{2b}$$

$$v_j^2 = v_i^2 - 2(\boldsymbol{r}_{ij}p_{ij} + \boldsymbol{x}_{ij}q_{ij}) + (\boldsymbol{r}_{ij}^2 + \boldsymbol{x}_{ij}^2)\frac{p_{ij}^2 + q_{ij}^2}{v_i^2} \tag{2c}$$

We use the relaxation of the DistFlow equations described by Farivar et al. [11]. The relaxation substitutes v_i^2 with the variable w_i which denotes the square of the voltage magnitude at each bus, and defines the variables l_{ij},

$$l_{ij} = \frac{p_{ij}^2 + q_{ij}^2}{w_i}$$

representing the square of the current magnitude on each line. This reduces the source of non-convexity into one equation, which is relaxed into an inequality. The overall relaxation is

$$p_{ij} = \sum_{k:(j,k)\in E} p_{jk} + \boldsymbol{r}_{ij}l_{ij} \tag{3a}$$

$$q_{ij} = \sum_{k:(j,k)\in E} q_{jk} + \boldsymbol{x}_{ij}l_{ij} \tag{3b}$$

$$w_j = w_i - 2(\boldsymbol{r}_{ij}p_{ij} + \boldsymbol{x}_{ij}q_{ij}) + (\boldsymbol{r}_{ij}^2 + \boldsymbol{x}_{ij}^2)l_{ij} \tag{3c}$$

$$l_{ij}w_i \geqslant p_{ij}^2 + q_{ij}^2 \tag{3d}$$

This relaxation was used by Farivar et al. [11] for Volt/VAR control; we will extend it to the case of reconfiguration. In solving a range of reconfiguration problems including loss minimization and load balancing, Taylor and Hover [7] used a second-order cone approximation of the DistFlow equations which ignores the last term in Eq. (2c). This approximation does not come with guarantees on the feasibility and quality of the solution returned. The alternative relaxation, proposed by Jabr et al. [15], will also be a point of comparison in our experimental section.

Meshed distribution networks

As the penetration of distributed generation increases, distribution networks will progressively evolve into active meshed networks with bi-directional power flows. Therefore, this paper also considers networks with general meshed topologies. The steady-state AC power flow equations become,

$$p_{ij} = \boldsymbol{g}_{ij}v_i^2 - \boldsymbol{g}_{ij}v_iv_j\cos(\theta_i - \theta_j) - \boldsymbol{b}_{ij}v_iv_j\sin(\theta_i - \theta_j) \tag{4a}$$

$$q_{ij} = -\boldsymbol{b}_{ij}v_i^2 + \boldsymbol{b}_{ij}v_iv_j\cos(\theta_i - \theta_j) - \boldsymbol{g}_{ij}v_iv_j\sin(\theta_i - \theta_j) \tag{4b}$$

Defining strong and scalable relaxations for these equations is less straightforward in this case. We use the recent convex quadratic relaxation proposed in [13], which offers a number of advantages over other relaxations [12,17]. In particular, SDP solvers required by the SDP relaxation [12] are less mature than nonlinear quadratic solvers in terms of scalability. Furthermore, it can be shown that [13] is strictly stronger than the SOCP relaxation discussed in [17] since the former contains a superset of constraints with respect to the latter.

Here, we only mention the main principles behind the relaxation and refer the reader to [13] for further details. As shown in Fig. 1, the formulation exploits convex relaxations of the nonlinear terms appearing in Eqs. (4a)–(4b). It uses:

- a quadratic relaxation of cosine (for $\boldsymbol{\theta}^u \leqslant \frac{\pi}{2}$)

$$\langle\cos(\theta)\rangle^R \equiv \begin{cases} \check{c}_{ij} & \leqslant \quad 1 - \frac{1-\cos(\boldsymbol{\theta}^u)}{(\boldsymbol{\theta}^u)^2}\theta^2 \\ \check{c}_{ij} & \geqslant \quad \cos(\boldsymbol{\theta}^u) \end{cases}$$

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