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Power system multi-parameter small signal stability analysis based on 2nd order perturbation theory

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ABSTRACT

A novel method based on multi-parameter 2nd order perturbation sensitivity is proposed to analyze the low-frequency oscillation modes in large-scale interconnected power system, since the low-frequency oscillation mode change is hard to determine due to the violent fluctuation of multiple parameters during operation. Firstly, the multi-parameter 2nd order perturbation sensitivity matrices of eigenvalues and eigenvectors are deduced. Then, their multi-parameter 2nd order estimated values are calculated. On the basis of this, the changing system oscillation modes under multiple parameters variation are estimated. The simulation results of WECC (Western Electricity Coordinating Council) system verify that this method is able to assess the small signal stability of the system relatively accurately even several parameters of the system change. Then it can adjust appropriate dispatching method accordingly to improve the damping of dominant oscillation mode. Also, this method makes the solving process direct and clear since it avoids the burdensome derivation calculation of 2nd order sensitivity, and it is time-saving by avoiding solving complicated high-dimensional state matrices.

Introduction

Due to the increasing scale of the power system as well as the changing and complex operation conditions, the safety and stability of the system has been seriously threatened by the often occurrence of low-frequency oscillation and small signal stability [1–5]. Thus, a thorough and explicit method to analyze and control small signal stability is in urgent need.

Currently, most small signal stability analysis methods are based on certainty theory. There are fewer researches considering uncertain parameters, mainly including the probability analysis method [6,7], the interval analysis method [8–10] and the sensitivity analysis method [11–13], etc. Probability analysis method uses probabilistic eigenvalue to demonstrate the influence of random factor on system stability by establishing a relationship between random parameters and state variables, and a relationship between the expectations and covariance of the state variables and eigenvalues. Interval analysis method uses interval distribution to model the small signal stability of the system under uncertain information by establishing a relationship between operation conditions and eigenvalues with uncertain information.

The sensitivity analysis method reflects the relationship between the change of system parameters and the change of eigenvalues. It can not only be used to analyze the dominant parameters that affect small signal stability, but also guide power output and parameter adjustment, providing operators with relatively comprehensive information of small signal stability. With small fluctuation of the operation parameters, the system can be approximately considered operating in the linear region. Thus, a result that is accurate enough can be calculated by 1st order eigenvalue sensitivity [14,15]. But when the fluctuation of the operation parameters is relatively large, the non-linear characteristics of the eigenvalues and system parameters begin to be exhibited. And the accuracy of the calculation based on 1st order eigenvalue becomes limited [16]. Meanwhile, multi-parameter fluctuation in large-scale interconnected system leads to the mode change of low-frequency oscillation as well as the oscillation mode. So the change of the system oscillation mode under large multi-parameter fluctuation needs urgent study.

Therefore, this paper uses matrix perturbation theory and Taylor expansion to deduce the multi-parameter 2nd order perturbation sensitivity matrix of the eigensolutions. Then, the multi-parameter 2nd order estimated value of eigensolutions is calculated. Then the mode change of system oscillation under

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Nomenclature

$\mathbf{A}$	the state matrix of the system
$\mathbf{A}_0$	the state matrix of original system
$\Delta\mathbf{A}$	the incremental matrix caused by the parameters change
$\mathbf{A}_s$	the perturbation sensitivity matrix of state matrix
$\Delta\mathbf{A}(\alpha^{(s)})$	the increment of $\mathbf{A}$ in terms of $\alpha^{(s)}$
$\mathbf{u}_i$	the corresponding right eigenvector of the ith eigenvalue
$\mathbf{u}_{ji}$	the jnd order perturbations of the right eigenvector
u_{jik}	the kth row of the jst order increment matrix of $\mathbf{u}_i(\boldsymbol{\alpha})$
λ_i	the ith eigenvalue
λ_{ji}	the jnd perturbations of the eigenvalue
$\boldsymbol{\alpha}$	the vector composed of certain part of uncertain parameters
$\Delta\boldsymbol{\alpha}$	the vector composed of the uncertain part of uncertain parameters
L	the dimension of $\boldsymbol{\alpha}$
$\boldsymbol{\alpha}_0$	the perturbation parameter's original value
$\mathbf{G}^{\mathbf{T}}_{\lambda_i}(\boldsymbol{\alpha}_0)$	the gradient matrices of $\lambda_i(\boldsymbol{\alpha})$
$\mathbf{G}^{\mathbf{T}}_{u_{ik}}(\boldsymbol{\alpha}_0)$	the gradient matrices of $u_{ik}(\boldsymbol{\alpha})$
$\mathbf{H}_{\lambda_i}(\boldsymbol{\alpha}_0)$	the Hessian matrices of $\lambda_i(\boldsymbol{\alpha})$
$\mathbf{H}_{u_{ik}}(\boldsymbol{\alpha}_0)$	the Hessian matrices of $u_{ik}(\boldsymbol{\alpha})$
$\tilde{\mathbf{G}}^{\mathrm{T}}_{\lambda i}$	the 1st order perturbation sensitivity matrix of eigenvalue
$\tilde{\mathbf{G}}^{\mathrm{T}}_{uik}$	the kth row of matrix $\mathbf{G}_{ui}$
$\mathbf{G}_{ui}$	the 1st order perturbation sensitivity matrix of the eigenvector
$\tilde{\mathbf{G}}^{\mathrm{T}}_{uik}$	the kth row of matrix $\mathbf{G}_{ui}$
$\mathbf{h}_1$	$L \times L$ order square matrix
$\mathbf{h}_2$	$L \times L$ order square matrix
$\mathbf{h}$	the 2nd order perturbation matrix of eigenvalue
$\tilde{\mathbf{H}}^{\mathrm{T}}_{\lambda i}$	the 2nd order perturbation sensitivity matrix of eigenvalue
$\mathbf{H}_{uik}$	the 2nd order perturbation matrix of the kth row of eigenvalue
$\tilde{\mathbf{H}}_{uik}$	the 2nd order perturbation sensitivity matrix of eigenvector
f_0	the real system frequency after perturbation
f_i	the ith estimated value of the frequency
f_{err}	the frequency error
ξ_0	the real system damping ratio after perturbation
ξ_i	the ith order estimated value of the damping ratio
ξ_{err}	the damping ratio error

multi-parameter fluctuation is estimated. The simulation results of 127-bus equivalent of the Western Electricity Coordinating Council (WECC) system verify the validity and practicability of this method.

Sensitivity analysis based on matrix perturbation theory

In the power system, the matrix eigenvalue is described as:

$$\mathbf{A}\mathbf{u}_i = \lambda_i \mathbf{u}_i \tag{1}$$

where $\mathbf{A}$ is state matrix of the system, λ_i is ith eigenvalue, $\mathbf{u}_i$ is corresponding right eigenvector of the ith eigenvalue.

After the system parameter change, the state matrix of the system changes as follows:

$$\mathbf{A} = \mathbf{A}_0 + \Delta\mathbf{A} \tag{2}$$

where $\mathbf{A}_0$ is the state matrix of original system, $\Delta\mathbf{A}$ is the incremental matrix caused by the parameters change.

From the Taylor expansion, the matrix increment caused by system parameters change is expressed as:

$$\Delta\mathbf{A} = \sum_{s=1}^{L} \mathbf{A}_s \Delta\alpha^{(s)} \tag{3}$$

$$\begin{cases} \boldsymbol{\alpha} = [\alpha^{(1)}, \alpha^{(2)}, \cdots, \alpha^{(L)}]^{\mathbf{T}} \\ \Delta\boldsymbol{\alpha} = [\Delta\alpha^{(1)}, \Delta\alpha^{(2)}, \cdots, \Delta\alpha^{(L)}]^{\mathbf{T}} \end{cases} \tag{4}$$

where $\mathbf{A}_s = \frac{\Delta\mathbf{A}(\alpha^{(s)})}{\Delta\alpha^{(s)}}$ is perturbation sensitivity matrix of state matrix. $\Delta\mathbf{A}(\alpha^{(s)})$ is the increment of $\mathbf{A}$ in terms of $\alpha^{(s)}$. $\boldsymbol{\alpha}$ is the vector composed of certain part of uncertain parameters. $\Delta\boldsymbol{\alpha}$ is the vector composed of the uncertain part of uncertain parameters. L is the dimension of $\boldsymbol{\alpha}$.

Perform 2nd order Taylor expansion of $\lambda_i(\boldsymbol{\alpha})$, which is the ith eigenvalue, at perturbation parameter's original value $\boldsymbol{\alpha}_0$:

$$\lambda_i(\boldsymbol{\alpha}) = \lambda_i(\boldsymbol{\alpha}_0) + \mathbf{G}^{\mathbf{T}}_{\lambda_i}(\boldsymbol{\alpha}_0)\Delta\boldsymbol{\alpha} + \frac{1}{2}\Delta\boldsymbol{\alpha}^T \mathbf{H}_{\lambda i}(\boldsymbol{\alpha}_0)\Delta\boldsymbol{\alpha}, i = 1, \cdots, n \tag{5}$$

Meanwhile, perform 2nd order Taylor expansion of $u_{ik}(\boldsymbol{\alpha})$, the kth parameters of eigenvector $\mathbf{u}_i(\boldsymbol{\alpha})$, which is the corresponding eigenvector of $\lambda_i(\boldsymbol{\alpha})$, at $\boldsymbol{\alpha}_0$ as well:

$$u_{ik}(\boldsymbol{\alpha}) = u_{ik}(\boldsymbol{\alpha}_0) + \mathbf{G}^{\mathbf{T}}_{u_{ik}}(\boldsymbol{\alpha}_0)\Delta\boldsymbol{\alpha} + \frac{1}{2}\Delta\boldsymbol{\alpha}^T \mathbf{H}_{u_{ik}}(\boldsymbol{\alpha}_0)\Delta\boldsymbol{\alpha} \tag{6}$$

where $\mathbf{G}^{\mathbf{T}}_{\lambda_i}(\boldsymbol{\alpha}_0)$ and $\mathbf{G}^{\mathbf{T}}_{u_{ik}}(\boldsymbol{\alpha}_0)$ are the gradient matrices of $\lambda_i(\boldsymbol{\alpha})$ and $u_{ik}(\boldsymbol{\alpha})$ expressed in terms of $\boldsymbol{\alpha}$ respectively. $\mathbf{H}_{\lambda i}(\boldsymbol{\alpha}_0)$ and $\mathbf{H}_{u_{ik}}(\boldsymbol{\alpha}_0)$ are the Hessian matrices of $\lambda_i(\boldsymbol{\alpha})$ and $u_{ik}(\boldsymbol{\alpha})$ expressed in terms of $\boldsymbol{\alpha}$, respectively.

From the direct derivation method, the gradient matrix of $\lambda_i(\boldsymbol{\alpha})$ and $u_{ik}(\boldsymbol{\alpha})$ expressed in terms of $\boldsymbol{\alpha}$ is written as:

$$\begin{cases} \mathbf{G}^{T}_{\lambda_i}(\boldsymbol{\alpha}_0) = \left[\frac{\partial\lambda_i(\boldsymbol{\alpha})}{\partial\alpha^{(1)}} \; \frac{\partial\lambda_i(\boldsymbol{\alpha})}{\partial\alpha^{(2)}} \cdots \frac{\partial\lambda_i(\boldsymbol{\alpha})}{\partial\alpha^{(L)}}\right] \\ \mathbf{G}^{T}_{u_{ik}}(\boldsymbol{\alpha}_0) = \left[\frac{\partial u_{ik}(\boldsymbol{\alpha})}{\partial\alpha^{(1)}} \; \frac{\partial u_{ik}(\boldsymbol{\alpha})}{\partial\alpha^{(2)}} \cdots \frac{\partial u_{ik}(\boldsymbol{\alpha})}{\partial\alpha^{(L)}}\right] \end{cases} \tag{7}$$

From the direct derivation method, the Hessian matrix of $\lambda_i(\boldsymbol{\alpha})$ and $u_{ik}(\boldsymbol{\alpha})$ expressed in terms of $\boldsymbol{\alpha}$ is written as:

$$\begin{cases} \mathbf{H}_{\lambda_i}(\boldsymbol{\alpha}_0) = \begin{bmatrix} \frac{\partial^2\lambda_i(\boldsymbol{\alpha})}{\partial\alpha^{(1)}\partial\alpha^{(1)}} & \cdots & \frac{\partial^2\lambda_i(\boldsymbol{\alpha})}{\partial\alpha^{(1)}\partial\alpha^{(L)}} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2\lambda_i(\boldsymbol{\alpha})}{\partial\alpha^{(L)}\partial\alpha^{(1)}} & \cdots & \frac{\partial^2\lambda_i(\boldsymbol{\alpha})}{\partial\alpha^{(L)}\partial\alpha^{(L)}} \end{bmatrix} \\ \mathbf{H}_{u_{ik}}(\boldsymbol{\alpha}_0) = \begin{bmatrix} \frac{\partial^2 u_{ik}(\boldsymbol{\alpha})}{\partial\alpha^{(1)}\partial\alpha^{(1)}} & \cdots & \frac{\partial^2 u_{ik}(\boldsymbol{\alpha})}{\partial\alpha^{(1)}\partial\alpha^{(L)}} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 u_{ik}(\boldsymbol{\alpha})}{\partial\alpha^{(L)}\partial\alpha^{(1)}} & \cdots & \frac{\partial^2 u_{ik}(\boldsymbol{\alpha})}{\partial\alpha^{(L)}\partial\alpha^{(L)}} \end{bmatrix} \end{cases} \tag{8}$$

In the power system, the gradient matrix and Hessian matrix cannot be obtained by the derivation method directly since the eigenvalue and eigenvector are implicit functions of the system parameters. Therefore, the analysis of low-frequency oscillation modes of the power system by the matrix perturbation theory is proposed in this paper.

From the matrix perturbation theory [17,18], the 2nd order estimated values of the eigenvalue and its corresponding eigenvector after system parameter change is written as: