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Analysis of the radial operation of distribution systems considering operation with minimal losses

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A R T I C L E I N F O

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A B S T R A C T

Electric power distribution systems, and particularly those with overhead circuits, operate radially but as the topology of the systems is meshed, therefore a set of circuits needs to be disconnected. In this context the problem of optimal reconfiguration of a distribution system is formulated with the goal of finding a radial topology for the operation of the system. This paper utilizes experimental tests and preliminary theoretical analysis to show that radial topology is one of the worst topologies to use if the goal is to minimize power losses in a power distribution system. For this reason, it is important to initiate a theoretical and practical discussion on whether it is worthwhile to operate a distribution system in a radial form. This topic is becoming increasingly important within the modern operation of electrical systems, which requires them to operate as efficiently as possible, utilizing all available resources to improve and optimize the operation of electric power systems. Experimental tests demonstrate the importance of this issue.

Introduction

An electrical distribution system (EDS) is the part of the electrical system that is physically located in the area of consumption, i.e., that portion of the electrical system that starts at the distribution substation. Such system need adequate planning so that it can operate efficiently and reliably. Among the operation and expansion planning problems of distribution systems are the problems of reconfiguration of radial distribution systems and the problem of expansion planning of distribution systems. Both problems require that the optimal solution found be a radial topology.

In this paper, we only analyze the problem of reconfiguration of distribution systems (RDS) but our conclusions can be extended to the distribution system expansion planning problem [1–3]. In the RDS problem, the purpose is usually to find a radial topology that allows the system to operate with minimal power losses. Radial topology was adopted by distribution companies some decades ago as the appropriate topology for operating a distribution system and especially for distribution systems with overhead circuits.

The RDS problem is a complex problem related to planning the operation of distribution systems. For various technical reasons, there is a prevailing paradigm that a distribution system should operate in a radial configuration although the system has a meshed structure. The purpose is to find a radial topology that optimizes one or more objectives. The most common objective is the minimization of power losses. The EDS can be viewed as a graph with nodes and arcs, in which case, the purpose is to find the tree of the graph that allows the system to operate with minimal power losses, that is, a radial topology (tree) must be found that allows the system to operate with minimal losses and in an adequate way in relation to other operational constraints. This is a mixed integer nonlinear programming problem that is very difficult to solve. The technical reasons used to justify the radial operation of an EDS, include two reasons that are mentioned very often in the literature: (a) radial topology makes it possible to improve the coordination of protection and, (b) radial topology reduces short-circuit currents in substations.

In the specialized literature there are many optimization techniques to solve the RDS problem. They can be divided into two major groups: (1) exact techniques, and (2) heuristics and metaheuristics. Exact techniques, such as branch-and-bound algorithms, were initially used only for relaxed models or using linearization techniques [1,17]. After complete models of these problems started to be used, heuristics and metaheuristics have been used very successfully to solve these problems in the last decades [4–16,18–20]. Some references regarding solution techniques for solving the RDS problem can be found in [1–13]. An important aspect of mathematical modeling is the efficient representation of the radiality constraint. For a long time, this was a controversial topic but in [2], a mathematical model that adequately represents

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the radiality constraint was presented along with the respective proof.

In this study, an initial critique of to the paradigm that stipulates that the EDS must operate radially is presented. There is no accurate information about when this became the paradigm of operation but it was probably in the 1950s and 1960s. At that time, the application of optimization techniques to the operation of power systems was very limited. For example, an important 1975 article by Merlin–Back [1], notes that: "This paper proposes a method for determining a minimum-loss operating configuration in a meshed distribution system to be operated in spanning tree structure (the usual case for a distribution system)". Therefore, it can be concluded that by 1975, the paradigm of radial operation was already fully established.

This paradigm has not been questioned even in the most recent papers. In [23] for example, the authors argue that distribution systems have a meshed structure but are usually operated as radial to assure effective coordination of their protection systems under emergency conditions and operational failures. The paper presents two new optimization models for the RDS problem: a mixed-integer conic programming model and a mixed-integer linear programming model. A radiality constraint is applied in both models. In [24,25], the authors consider that a network should be radial but there is no explanation for this as it can be seen in this excerpt: "The first constraint is that in the case of final solution, the electrical network should be a completely radial configuration. Both Kirchhoffs voltage and current laws must be observed to satisfy the AC load flow equations of the radial electrical system".

In [26], there is some indication that the authors agreed this paradigm, because, for them, an essential criterion for the RDS is the conservation of the radial nature of the network, primarily to facilitate the coordination of protection. However, according to the authors, the methodology of probabilistic reliability evaluation described in their paper is not limited to radial distribution networks but is also valid for meshed systems.

In this paper, we analyze the paradigm of radial operation and suggest new lines of research aimed at changing this paradigm. Experimental tests and a preliminary theoretical analysis show that radial topology is the worst operation topology if the goal is to operate the distribution system with minimal power losses. This observation, essentially experimental, is not compatible with modern proposals for the operation of electrical systems that propose optimal operation of all parts of the system.

This study is limited to analyzing the operation of a distribution system and the paradigm of operation in radial topology. In future studies, we will analyze other aspects of the operation, especially changes and additional costs that may be incurred by operation proposals different that radial topology. Some of this research will include consideration of the economic and operating implications of non-radial operation on the increase of short-circuit currents in the substation and additional costs in the coordination of protection.

The contributions of this paper are:

1. An experimental analysis that proves that radial topology is one of the worst topologies if the goal is for the system to operate with minimal power losses.
2. A preliminary theoretical analysis that shows that the initial radial topology is not the most appropriate if the goal is for the system to operate with minimal losses.
3. The presentation of a mathematical model and an optimization proposal that permits the analysis of the influence of a distribution system topology on power losses.
4. The initiation of a debate in the scientific community in order to consolidate or change the paradigm of operation of distribution systems, taking into account that a topology with a small number of loops can be interesting.

Mathematical modeling of the RDS problem

We consider that traditional mathematical modeling of the RDS problem allows finding a radial topology with minimal losses. Thus, according to [2], the RDS problem for one substation is modeled as follows:

$$\min\ v = \sum_{(ij)\in\Omega_l}\left[g_{ij}x_{ij}\left(V_i^2 + V_j^2 - 2V_iV_j\cos\theta_{ij}\right)\right] \tag{1}$$

s.t.

$$P_{S_i} - P_{D_i} - \sum_{j\in\Omega_{b_i}}(x_{ij}P_{ij}) = 0 \quad \forall i\in\Omega_b \tag{2}$$

$$Q_{S_i} - Q_{D_i} - \sum_{j\in\Omega_{b_i}}(x_{ij}Q_{ij}) = 0 \quad \forall i\in\Omega_b \tag{3}$$

$$\underline{V} \leqslant V_i \leqslant \overline{V} \quad \forall i\in\Omega_b \tag{4}$$

$$I_{r_{ij}}^2 + I_{m_{ij}}^2 \leqslant \bar{I}_{ij}^2 x_{ij} \quad \forall (i,j)\in\Omega_l \tag{5}$$

$$x_{ij}\in\{0,1\} \quad \forall (i,j)\in\Omega_l \tag{6}$$

$$\sum_{(ij)\in\Omega_l} x_{ij} = n_b - 1 \tag{7}$$

in which Ω_l is the set of circuits; Ω_b is the set of nodes; Ω_{b_i} is the set of connected nodes in node i ($\Omega_{b_i}\subset\Omega_b$); $\underline{V}$ is the minimum voltage magnitude; $\overline{V}$ is the maximum voltage magnitude; $\bar{I}_{ij}$ is the maximum current flow in circuit ij; n_b is the number of nodes ($n_b = |\Omega_b|$); P_{D_i} is the active power demand at node i; Q_{D_i} is the reactive power demand at node i; g_{ij} is the conductance of circuit ij; b_{ij} is the susceptance of circuit ij; P_{ij} is the active power flow that leaves node i toward node j; Q_{ij} is the reactive power flow that leaves node i toward node j; $I_{r_{ij}}$ is the real component of the current flow in circuit ij; $I_{m_{ij}}$ is the imaginary component of the current flow in circuit ij; v represents the operational losses of the EDS; x_{ij} is the binary variable that determines if the circuit between nodes i and j is connected; V_i is the voltage magnitude at node i; θ_{ij} is the difference of phase angle between nodes i and j; P_{S_i} is the active power provided by substation at node i; Q_{S_i} is the reactive power provided by substation at node i;

Furthermore, in the modeling above, the variable elements P_{ij} and Q_{ij} are obtained by (8) and (9).

$$P_{ij} = V_i^2 g_{ij} - V_iV_j(g_{ij}\cos\theta_{ij} + b_{ij}\sin\theta_{ij}) \tag{8}$$

$$Q_{ij} = -V_i^2 b_{ij} - V_iV_j(g_{ij}\sin\theta_{ij} - b_{ij}\cos\theta_{ij}) \tag{9}$$

The objective function (1) represents the active power losses in the operation of the EDS. Constraints (2) and (3), along with (8) and (9), represent constraints related to Kirchhoff's Laws for the AC model. Additionally, (4) represents the operational constraints on the voltage magnitude of nodes. The real and reactive components of the current in circuit ij in (5) are given by (10) and (11), respectively.

$$I_{r_{ij}} = g_{ij}(V_i\cos\theta_i - V_j\cos\theta_j) - b_{ij}(V_i\sin\theta_i - V_j\sin\theta_j) \tag{10}$$

$$I_{m_{ij}} = g_{ij}(V_i\sin\theta_i - V_j\sin\theta_j) + b_{ij}(V_i\cos\theta_i - V_j\cos\theta_j) \tag{11}$$

The constraints presented in (6) represent the decision variables in the optimization problem and also account for the complexity of the problem given that x_{ij} are binary. Thus, for a feasible solution of the problem (1)–(11), the circuit between nodes i and j is connected if $x_{ij} = 1$ and is not connected if $x_{ij} = 0$. Additionally, only in substation P_{S_i} and Q_{S_i} do they have non-zero values. Finally (7) indicates that any feasible solution of the problem (1)–(11) must have exactly $(n_b - 1)$ circuits connected to the EDS. A detailed analysis of this condition is one of the main topics of this study.