Electrical Power and Energy Systems 64 (2015) 476–482

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Research on time-delay stability upper bound of power system wide-area damping controllers based on improved free-weighting matrices and generalized eigenvalue problem

Jing Ma *, Junchen Li, Xiang Gao, Zengping Wang

State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources, North China Electric Power University, Beijing 102206, China

ARTICLE INFO

Article history:
Received 25 July 2013
Received in revised form 1 July 2014
Accepted 17 July 2014

Keywords:
Improved free-weighting matrices
L–K functional
Time-delay upper bound
Generalized eigenvalue problem

ABSTRACT

A novel method is proposed in this paper to determine the time-delay stability upper-bound of the power system with wide-area damping controllers based on the improved free-weighting matrix and Generalized Eigenvalue Problem (gevp). First, a new class of Lyapunov–Krasovskii functional is constructed and its derivative function along the system is gained. Second, necessary loose items are added to the derivative function to reduce conservativeness, and the time-delay stability criterion based on the improved free-weighting matrices is formed. And then the time-delay stability criterion is equivalently transformed to a generalized eigenvalue problem. Thus by solving the gevp, the time-delay stability upper bound of the system is gained. Time-domain simulation tests on the IEEE 4-machine 11-bus system and IEEE 16-machine 68-bus system verify the correctness and effectiveness of the method.

Introduction

In view of the time-delay problem in feedback controller design based on wide area information, which may affect the control effect, even leading to negative damping [1–5], research on the time-delay stability upper bound of the system is of great need.

The current research methods can be sorted into three categories: time-domain method, frequency-domain method and direct method. The time domain method is able to determine the stability of the system under certain circumstances. However, further research is required concerning the acquisition of information such as the stability degree and the time-delay stability upper bound. The frequency-domain method is able to reveal the variation characteristic of the time-delay system to some extent by seeking the key eigenvalue of the system in the real space. However, the calculation speed is low due to large computation. Based on the Lyapunov theory and the Linear Matrix Inequality (LMI) technique, the direct method is able to take into account both the time-delay randomness and the switch link, thus can be more widely applied. However, the direct method is relatively conservative and much study has been conducted on how to reduce its conservativeness [6–11].

For the time-delay system stability problem, the Lyapunov–Krasovskii functional is widely used due to its consideration for the influence of the system past on the system variation rate. The main idea of the Lyapunov–Krasovskii functional is as follows. First, a positive definite functional containing explicit time-delay is constructed. Second, the derivative of the functional along the system trajectory is calculated. And then, according to the Krasovskii stability theorem, the sufficient condition of the system stability criterion could be obtained. Ref. [12] by Park et al. and Refs. [13,14] use the improved Lyapunov–Krasovskii functional that contains multiple integral items, the result of which is less conservative. However, the multiple integral items also add to the difficulty in dealing with the derivative of the functional. More inequalities will be needed to bound the cross terms in the derivative of the Lyapunov–Krasovskii functional, which will limit the reduction of conservativeness. Refs. [15,16] use the delay-partitioning approach to partition the time-delay region into $N+1$ sub-regions, and then use a sub-optimization method to determine the time-delay partition parameters, so that the conservativeness could be reduced. However, this method greatly adds to the computational complexity, with a very limited reduction in conservativeness. Refs. [17,18] apply the integral inequality method to time-delay stability analysis, which is simple in form with fewer matrix variables, and is easy for theoretical analysis and calculation. However, the conservativeness is often widely divergent due to different amplification degrees of different methods.

* Corresponding author. Address: No. 52 Mailbox, North China Electric Power University, No. 2 Beinong Road, Changping District, Beijing 102206, China. Tel.: +86 10 80794899 (Work), mobile: +86 15801659769.

E-mail address: hdmajing@ncepu.edu.cn (J. Ma).

http://dx.doi.org/10.1016/j.ijepes.2014.07.052
0142-0615/© 2014 Published by Elsevier Ltd.

Nomenclature

$\boldsymbol{x}$	system state vector	h	time-delay stability upper bound
$\boldsymbol{u}$	control input vector	$\mathbf{P}$	symmetric positive definite matrix
$\boldsymbol{y}$	output vector	$\mathbf{Q}$	symmetric positive definite matrix
$\boldsymbol{A}$	system state matrix	$\mathbf{Z}_i$	symmetric positive definite matrix
$\boldsymbol{B}$	control input matrix	$\mathbf{N}$	free weighting matrix
$\boldsymbol{C}$	output matrix	$\mathbf{S}$	free weighting matrix
$\mathbf{K}$	control matrix	$\mathbf{M}$	free weighting matrix
$\boldsymbol{A}_d$	time-delay matrix	$\mathbf{I}$	unit matrix
$d(t)$	time delay	$\mathbf{Y}_i$	supplementary matrix

In order to further reduce the conservativeness of the direct method, a novel method is proposed in this paper to determine the time-delay stability upper-bound of the power system wide-area damping controllers based on the improved free-weighting matrix and Generalized Eigenvalue Problem (gevp). First, a new class of Lyapunov–Krasovskii functional is constructed and its derivative function along the system is gained. Second, necessary loose items are added to the derivative function to reduce conservativeness, and the time-delay stability criterion based on the improved free-weighting matrices is formed. And then the time-delay stability criterion is equivalently transformed to a Generalized Eigenvalue Problem (gevp), thus by solving the gevp the time-delay stability upper bound of the system is gained. Time-domain simulation tests on the IEEE 4-machine 11-bus system and IEEE 16-machine 68-bus system verify the correctness and effectiveness of the method.

Time delay model of the power system

The state equation of a multi-input multi-output power system is

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{u}(t) = \mathbf{K}\mathbf{x}(t) \end{cases} \tag{1}$$

where $\mathbf{x} \in \mathbf{R}^n$ is the system state vector, $\mathbf{u} \in \mathbf{R}^m$ is the control input vector, $\mathbf{A} \in \mathbf{R}^{n\times n}$ is the system state matrix, $\mathbf{B} \in \mathbf{R}^{n\times m}$ is the system control matrix. When there are different types of supplementary controllers in the system, the control information is all included in $\mathbf{B}$.

The closed-loop system after the state feedback is:

$$\dot{\mathbf{x}}(t) = \mathbf{C}\mathbf{x}(t) \tag{2}$$

where $\mathbf{C}$ is the closed-loop state matrix. $\mathbf{C} = \mathbf{A} + \mathbf{BK}$, where $\mathbf{B} \in \mathbf{R}^{n\times m}$ is the integrated state feedback matrix of different supplementary controllers.

In actual power system, the control input vector is transmitted to the controllers via SCADA/WAMS. During the signal transmission, certain time delay is unavoidable. Thus, the corresponding closed-loop system can be described as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{BK}\mathbf{x}(t - d(t)) \tag{3}$$

Seen from (3), the time-delay matrix of the power system is $\mathbf{A_d} = \mathbf{BK}$. For system with time-delay links, the state equation takes the following form:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{A}_d\mathbf{x}(t - d(t)), \ t > 0 \\ \mathbf{x}(t) = \boldsymbol{\varphi}(t), \ t \in [-h, 0] \end{cases} \tag{4}$$

where $\mathbf{x}(t) \in \mathbf{R}^n$ is the state vector. $\mathbf{A}$ and $\mathbf{A_d}$ are the state matrix and time-delay matrix respectively. h is the time-delay stability upper bound.

In (4), the time delay $d(t)$ meets inequalities (5) and (6):

$$0 \leqslant d(t) \leqslant h \tag{5}$$

$$\dot{d}(t) \leqslant \mu \tag{6}$$

Time delay stability criterion

For the time-delay system stability problem, the Lyapunov–Krasovskii functional is widely used due to its consideration for the influence of the system past on the system variation rate. Early time-delay system stability research is focused on the time-delay irrelevant type, i.e. the system stability is not related with whether there is time delay in the system, and the Lyapunov–Krasovskii functional selected is usually:

$$\begin{cases} \mathbf{V}'(\mathbf{x}) = \mathbf{V}_1'(\mathbf{x}) + \mathbf{V}_2'(\mathbf{x}) \\ \mathbf{V}_1'(\mathbf{x}) = \mathbf{x}^{\mathbf{T}}(t)\mathbf{P}\mathbf{x}(t) \\ \mathbf{V}_2'(\mathbf{x}) = \int_{t-d}^{t} \mathbf{x}^{\mathbf{T}}(s)\mathbf{Q}\mathbf{x}(s)ds \end{cases} \tag{7}$$

where $\mathbf{x}(t)$ is the system state vector, d is a constant time delay. $\mathbf{P}$ and $\mathbf{Q}$ are positive definite symmetrical matrix variables to be determined.

Calculate the derivative of (7) concerning t, so that the following stability criterion can be gained:

$$\begin{bmatrix} \mathbf{PA} + \mathbf{A}^{\mathbf{T}}\mathbf{P} + \mathbf{Q} & \mathbf{PI} \\ \mathbf{I}^{\mathbf{T}}\mathbf{P} & -\mathbf{Q} \end{bmatrix} < 0 \tag{8}$$

where $\mathbf{I}$ is the unit matrix.

The time-delay irrelevant stability criterion in (8) does not contain time-delay relevant information. For systems with small time delay, the criterion is very conservative. For systems with relatively big time delay, the criterion usually fails to determine the system stability correctly. The deficiencies of the time-delay irrelevant stability criterion greatly boost the research on time-delay relevant stability. In such research, the maximum time-delay upper bound that guarantees system stability is the main index to evaluate the conservativeness of the time-delay relevant conditions. And a quadratic form double integral item $\mathbf{V}_3'(\mathbf{x})$ is added to the Lyapunov–Krasovskii functional shown in (7), i.e.

$$\begin{cases} \mathbf{V}'(\mathbf{x}) = \mathbf{V}_1'(\mathbf{x}) + \mathbf{V}_2'(\mathbf{x}) + \mathbf{V}_3'(\mathbf{x}) \\ \mathbf{V}_1'(\mathbf{x}) = \mathbf{x}^{\mathbf{T}}(t)\mathbf{P}\mathbf{x}(t) \\ \mathbf{V}_2'(\mathbf{x}) = \int_{t-d}^{t} \mathbf{x}^{\mathbf{T}}(s)\mathbf{Q}\mathbf{x}(s)ds \\ \mathbf{V}_3'(\mathbf{x}) = \int_{-d}^{0}\int_{t+\theta}^{t} \dot{\mathbf{x}}^{\mathbf{T}}(s)Z\dot{\mathbf{x}}(s)ds\, d\theta \end{cases} \tag{9}$$

where $\boldsymbol{Z}$ is a positive definite symmetrical matrix variable to be determined.

According to the Lyapunov–Krasovskii stability theorem, the time-delay relevant stability criterion of (9) could be obtained. However, how to deal with the quadratic form double integral item resulting from the derivation is a critical problem which could affect the conservativeness of the time-delay relevant stability criterion. The free weighting matrix method is able to deal with the