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A distributed state estimation method for power systems incorporating linear and nonlinear models

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ABSTRACT

High-voltage transmission networks are commonly equipped with phasor measurement units (PMU), and some of them are PMU observable. However, PMUs are seldom installed in distribution networks due to budget limitations. The state estimation equations of PMU observable areas are linear, while those of other areas remain nonlinear. This paper proposes a new distributed state estimation method for solving multi-area state estimation problems, in which linear models are used for high-voltage transmission network, while nonlinear models are adopted for other areas. In PMU observable area, we select coordinating variables as generalized cost functions which accurately represent the sensitivity between the linear SE objective function and boundary states. Consequently, SE results identical to a centralized estimator can be obtained without iterations at the coordination level. This paper presents the problem model and theoretical analysis of the proposed method, and shows its effectiveness by numerical tests.

Introduction

PMUs offer accurate phasor measurements and are very helpful for improving the accuracy of state estimation [1,2]. PMUs are increasingly being installed in high-voltage transmission systems, and most 500 kV and higher level transmission networks in China are already PMU observable. More transmission networks worldwide will become PMU observable in the future. When the transmission networks are PMU observable, its state variables can be estimated firstly by traditional nonlinear estimator with RTU measurements. After that, these estimated state variables together with PMU voltage and current measurements are considered to be a new measurement set. In such a new measurement set, each measurement is a linear function to the state variables and can be modeled as linear estimator [3–5]. Consequently, the robustness and efficiency of SE can be significantly enhanced. On the other hand, distribution networks are not PMU observable and their SE models are still nonlinear. In reality, the transmission and distribution networks are separately monitored by transmission network operator (TNO) and distribution network operators (DNO). It is therefore a technical challenge to obtain global optimal SE results for such an overall interconnected power system based on the distributed state estimators located in different control centers.

Considerable researches have been published on distributed state estimation (DSE) methods for multi-area power systems. A critical survey of early DSE studies is presented in [6] and a recent taxonomy has been described in [7]. All DSE methods can be divided into two categories according to the type of boundary chosen: methods that use border buses as the boundary [8–12] or use tie lines as the boundary [13–15]. A challenge for methods in the first category is how to handle border injection measurements [8,9]. Because SE of each area is calculated separately, mismatches will appear on the boundary. A DSE method must eliminate these boundary mismatches. Moreover, an optimal DSE method will produce results identical to the results of CSE. An optimal DSE method usually requires iterations at the coordination level, and the fixed-point iterative technique or one of its improved versions is commonly used [11,15,16]. However, it is difficult to guarantee the convergence of such iterations [6,7]. Another type of DSE methods are non-optimal, which only eliminate boundary mismatches, but offer different results with CSE [10,17,18].

The extensive deployment of PMUs has led to many studies on their usage to enhance the accuracy and robustness of SE. An effective method for integrating PMU measurements into an existing SE program is proposed in [19]. PMU measurements are also used to coordinate the voltage phase angles of different areas in DSE [17,20].

The main contribution of this paper is to propose a new DSE method for interconnected power systems incorporating PMU observable and unobservable areas. The proposed method uses a

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Nomenclature

SE	state estimation
PMU	phasor measurement unit
RTU	remote terminal unit
WAMS	wide area measurement system
SCADA	supervisory control and data acquisition
TNO	transmission network operator
DNO	distribution network operator
DSE	distributed state estimation
CSE	centralized State Estimator
$\boldsymbol{x}$	power system state variable vector
$\boldsymbol{z}$	measurement value vector
$\boldsymbol{h}$	measurement function
$\boldsymbol{f}$	weighted least square objective function
$\boldsymbol{r}$	residual vector
$\boldsymbol{W}$	diagonal measurement weight matrix
$\boldsymbol{H}$	measurement Jacobian matrix
$\boldsymbol{g}$	gradient vector, $\boldsymbol{g} = \boldsymbol{H}^{\mathrm{T}}\boldsymbol{Wr}$, superscript T means transposition.
$\boldsymbol{G}$	information matrix, $\boldsymbol{G} = \boldsymbol{H}^{\mathrm{T}}\boldsymbol{WH}$
$\boldsymbol{e}$	measurement errors
ψ	complex number transformation from polar coordinate to rectangular
$\boldsymbol{J}$	Jacobian matrix of ψ

linear SE model with PMU measurements for PMU observable areas and nonlinear SE models with RTU measurements for the rest areas. The coordination variable is chosen as a generalized cost function, which is composed of a low-dimension matrix and vector without any privacy. Since the generalized cost function accurately represents the sensitivity between the optimality for the SE in the linear part and the boundary states, the proposed DSE method can achieve global optimality without iterations at the coordination level. On contrary, traditional DSE methods usually use partial information near the boundary for coordination. As a result, these methods either need iterations at coordination level [11,15,16] or non-optimal [4], even if there is a linear SE part. The generalized cost function can be calculated conveniently by the efficient Gaussian elimination.

This paper is organized as follows. A simple two areas model is described in Section 'Problem formulation', and is used to derive the DSE method in Section 'Proposed method'. In Section 'Modeling with multiple areas', the proposed model is extended to multiple areas. Comparison with traditional DSE methods, gross error identification, computational burden and data exchange are discussed in Section 'Discussion'. In Section 'Numerical tests', numerical tests are used to verify the performance of the proposed method.

Problem formulation

Without loss of generality, the global system is shown in Fig. 1 which consisted by two areas: a PMU observable area and an RTU observable area. These two areas are connected by tie lines, and the terminal buses of the tie lines are boundary buses with state variables denoted by $\boldsymbol{x}_B$. The boundary bus set B can be subdivided into the bus sets BL and BN, composed of boundary buses with and without PMU measurements, respectively. Since the boundaries are composed of tie-lines instead of boundary buses, boundary measurements can be divided to linear or nonlinear area clearly: PMU measurements distribute inside the linear area, the voltage and injections for bus BL, and the BL side tie-line measurements. Note that if PMUs are installed in buses in set BL, then the buses in BN are also PMU observable.

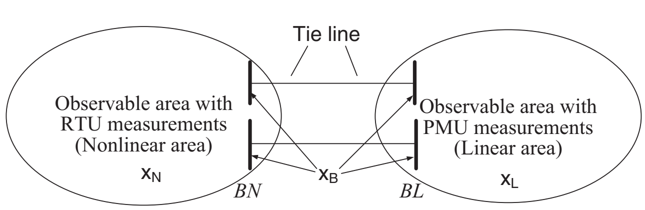

Fig. 1. Power system with two areas.

The optimal solution to the state estimation problem for the overall system can be obtained from

$$\min_{\boldsymbol{x}_L, \boldsymbol{x}_B^{v\theta}, \boldsymbol{x}_N} \quad f_\Sigma = f_L(\boldsymbol{x}_L, \boldsymbol{x}_B^{ef}) + f_N(\boldsymbol{x}_B^{v\theta}, \boldsymbol{x}_N) \tag{1}$$

where f_Σ is the global objective function for the interconnected power system. Throughout this paper, subscript L and N denotes linear and nonlinear areas, and subscript B denotes boundary variables. In linear area, $\boldsymbol{x}_L$ should be expressed in rectangular coordinates, while the state variables $\boldsymbol{x}_N$ in the nonlinear area are used to express in polar coordinates. Accordingly, the state variables $\boldsymbol{x}_B$ for the boundary buses should be expressed in rectangular coordinates in f_L, and polar coordinates in f_N. $\boldsymbol{x}_B^{ef}$ and $\boldsymbol{x}_B^{v\theta}$ are different expressions for $\boldsymbol{x}_B$ in rectangular and polar coordinates, their relationship can be represented as a mapping ψ:

$$\begin{aligned} \boldsymbol{x}_B^{ef} &= \psi(\boldsymbol{x}_B^{v\theta}) \\ \boldsymbol{x}_B^{v\theta} &= \psi^{-1}(\boldsymbol{x}_B^{ef}) \end{aligned} \tag{2}$$

The formulation of ψ can be elaborated as:

$$\psi : \begin{cases} e = V\cos\theta \\ f = V\sin\theta \end{cases} \tag{3}$$

Then the first-order optimality condition for (1) is given by

$$\begin{cases} \frac{\partial f_\Sigma}{\partial \boldsymbol{x}_N} = \frac{\partial f_N}{\partial \boldsymbol{x}_N} = 0 \\ \frac{\partial f_\Sigma}{\partial \boldsymbol{x}_B^{v\theta}} = \frac{\partial f_N}{\partial \hat{\boldsymbol{x}}_B^{v\theta}} + \boldsymbol{J}\frac{\partial f_L}{\partial \hat{\boldsymbol{x}}_B^{ef}} = 0 \\ \frac{\partial f_\Sigma}{\partial \hat{\boldsymbol{x}}_L} = \frac{\partial f_L}{\partial \hat{\boldsymbol{x}}_L} = 0 \end{cases} \tag{4}$$

By directly solving (4), the optimal SE result $(\hat{\boldsymbol{x}}_L, \hat{\boldsymbol{x}}_B^{v\theta}, \hat{\boldsymbol{x}}_N)$ can be obtained. However, this is an integrated technique, and a practical distributed solution is needed to get the same result in a more convenient manner.

Proposed method

Generalized cost function in the linear area

For the linear area, the measurement function can be expressed as

$$\boldsymbol{z}_L = \boldsymbol{H}_{LL}\boldsymbol{x}_L + \boldsymbol{H}_{LB}\boldsymbol{x}_B^{ef} + \boldsymbol{e}_L \tag{5}$$

where $\boldsymbol{H}_{LL}$ and $\boldsymbol{H}_{LB}$ are the constant measurement coefficient matrices for $\boldsymbol{x}_L$ and $\boldsymbol{x}_B^{ef}$, respectively.

The weighted least-squares solution of (5) can be directly obtained from