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Uncertainty analysis for bulk power systems reliability evaluation using Taylor series and nonparametric probability density estimation

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ABSTRACT

The uncertainties in reliability evaluation model are fundamentally classified into aleatory and epistemic types. Aleatory uncertainty arises from the intrinsic randomness associated with a physical system, such as components stochastic failure and repair process. Epistemic uncertainty, on the other hand, results from an incomplete or inaccurate scientific understanding of the underlying process, such as component reliability parameters uncertainty. It's significant for risk based decision to distinguish the two kinds of uncertainties and quantify their impacts on reliability analysis. In literatures, most of papers focused on aleatory uncertainty, and only a few of them discussed the epistemic uncertainty. This paper is aimed to address uncertainty analysis of reliability indices considering the randomness of reliability parameters. Two goals are achieved in this paper. Firstly, the reliability indices are approximated through Taylor series with high efficiency after component parameters change, and its accuracy is compared with rerunning reliability evaluation. Secondly, to uncover the uncertainty propagation from input reliability parameters level to reliability evaluation output level, two methods, i.e. Taylor series Approximation and Monte Carlo simulation combined with nonparametric probability density estimation are proposed. Results obtained for the RBTS and IEEE-RTS79 power systems are presented and the validity of the proposed methods is verified.

Introduction

Reliability evaluation of bulk power systems is a powerful tool to quantify the risk of electric service interruption incurred by uncontrollable and unpredictable failure events using uncertainty analysis theory [1–4]. Through the comprehensive quantitative analysis of the possibility and impact of the random failures, the probabilistic measures of interruption risk for system or delivery points can be identified by condensing contingency likelihood and severity into probabilistic risk indices. The purpose of the probabilistic risk evaluation for bulk power systems is to quantitatively assess the impacts of various kinds of random factors on power systems performance, and provide valuable reference information for risk management, risk control and risk based decision. Using the probabilistic risk evaluation technology, not only the coupling relation between the uncertain factors and system performance can be uncovered, but also the system bottlenecks and dominant random factors can be effectively revealed.

The reliability performance of a power system is often affected by unavoidable uncertainties, and probabilistic uncertainty analysis can quantify the effect of input random variables on the output results of reliability evaluation model. The uncertainty in reliability evaluation model is fundamentally classified into aleatory and epistemic types. Quantitative uncertainty analysis has become an integral and essential part of risk based design and decision making, and the clear distinction of these two kinds of uncertainties is useful for taking the reliability/risk informed decisions with confidence [5–9]. The problem of acknowledging and treating uncertainty is vital for practical usability of reliability analysis results because the reliability indices mixing both the uncertainties means that one cannot see how much of the total uncertainty comes from epistemic and aleatory uncertainties.

Aleatory uncertainty is also termed in the literature as objective, irreducible, inherent, and stochastic uncertainty. It describes the inherent randomness (variation) associated with a physical system or environment, such as failure and repair time of equipments in a power system. This type of uncertainty cannot be reduced or eliminated because it is an intrinsic nature of the system itself. Aleatory uncertainty is dealt with by probability theory. Given the failure logic of system and probability density functions of failure and repair time, sequential Monte Carlo simulation can be used to obtain the probability density of reliability indices [10,11], while analytical approach and nonsequential Monte Carlo

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simulation can be used to get the point estimation(usually the expected value) of reliability indices.

However, both Monte Carlo simulation and analytical approach are built on a number of model parameters that are based on what is currently known about the physics of the relevant processes and the behavior of systems. The model parameters, such as failure rates and repair rates, are not exactly known because of deterioration or lack of data. This kind of uncertainty associated with state of knowledge, is referred as epistemic uncertainty. Epistemic uncertainty derives from some level of ignorance or incomplete information about a system, and it's reducible if more information is collected. Because of this reason, it is also termed as reducible, subjective, and model form uncertainty.

In the published literatures, aleatory uncertainty have been the main focus of bulk power systems reliability evaluation, and only few of them are involved in parameter uncertainty. The equipment reliability parameters are the input data in system risk evaluation and can be estimated or measured from historical failure statistics using parameter estimation technique. The correctness and accuracy of the historical data is critical in risk evaluation, as we know uncertainties or even errors in historical statistics are unavoidable. An essential task in the parameter estimation is to reduce the impacts of the uncertainties or errors and enhance the accuracy. The parameter estimation methods can be divided into point, interval and distribution estimations among which the distribution estimation is the most sophisticated and can offer a probability distribution function (PDF) of a parameter. Using the probability distribution of input data can greatly enhance the accuracy in system risk evaluation. It needs more calculation efforts and higher requirements for assessment techniques when probability distribution of input data are used, however, utilizing the approach presented in this paper this problem can be effectually solved with nearly no extra computational effort.

The impact of parameter uncertainty must be addressed if the analysis is to serve as a tool in the decision making process. To uncover the impact of parameter uncertainty on reliability evaluation results, two critical questions must be deeply explored.

- How do we characterize the uncertainty of equipment reliability parameters?
- How does the uncertainty propagate from parameter level to model output level?

There are several methods available in the literature to research parameter uncertainty propagation such as evidence theory [7], interval arithmetic [8], fuzzy arithmetic [9], classical probability theory, and so on. They are different from each other, in terms of characterizing the input parameter uncertainty and also in kind of propagation from parameter level to model output level. Because the most widely known and developed methods are available within the mathematics of probability theory, in this paper before the uncertainty analysis can be performed a description of model parameters uncertainty must be available, i.e., failure rate/repair rate are characterized by a probability distribution and then how they are propagated to the system level is investigated. So the above questions are turned into how to get the analytical expression of the reliability indices with respect to reliability parameters and how to obtain the PDF of reliability indices when PDFs of reliability parameters are given.

Through sensitivity analysis of reliability indices with respect to equipment failure and repair rates, the first order Taylor series of reliability indices at the mean values of component reliability parameters can be achieved [12,13]. So the reliability indices can be analytically calculated with high accuracy after reliability parameters are slightly changed. But the results using first order Taylor series have larger error if reliability parameters have significant changes. To improve the accuracy, the second-order partial differentials of reliability indices with respect to reliability parameters and then the second order Taylor series are deduced in this paper.

After the approximate analytical expressions of reliability indices are developed, how to obtain the PDFs of reliability indices assuming the PDFs of reliability parameters are known become a vital step for parameter uncertainty analysis. Using the second order Taylor series, the reliability indices can be regarded as random sums consisting of both linear and quadratic items of uncertain reliability parameters. To avoid the direct convolution of the PDFs of the reliability parameters, several methods are presented in literatures, such as saddlepoint approximation [6], Gram–Charlier's expansion [14], and characteristic function method [15]. However there is a common limitation among these methods that the constituent items in the random sum must be mutually independent random variables, which is true for first order Taylor series but invalid for second order Taylor expansion because the linear item and quadratic item are correlated with each other. Because of the above reason a new approach termed as nonparametric probability density estimation has been applied to solve this problem in this paper.

The aim of this work is to develop an efficient and accurate method, which is expected to have the approximate accuracy as Monte Carlo simulation depicted in section IV, but with much higher efficiency. The proposed method is detailed in the following section.

This paper is organized as follows. Section II gives the mathematical description of parameter uncertainties for bulk power systems reliability evaluation model. Section III describes the first and second order derivatives of reliability indices with respect to component reliability parameters. Section IV depicts the fundamental principle to evaluate probability distribution of reliability indices through nonparametric probability density estimation technique, and two methods are presented there, i.e. Taylor series approximation and Monte Carlo simulation. The study results and the effectiveness of the proposed approach are illustrated in Section V. The conclusion drawn from the analysis is provided in Section VI. Details are provided in Appendix.

Mathematical description of epistemic uncertainty

The reliability index Y can be expressed with the joint function of aleatory and epistemic uncertainties.

$$Y = R(\mathbf{U}, \mathbf{V}), \tag{1}$$

where

- **U** set of all epistemic uncertainties (uncertain reliability parameters known as failure rate $\lambda$and repair rate μ),
- **V** set of all aleatory uncertainties (stochastic variables associated with component failure or repair, i.e. the time to failure and the time to repair),
- R computational model for reliability evaluation considered as a deterministic function of both uncertainties mentioned above,
- Y reliability index.

R represents the computational model which describes the functional relationship between reliability index Y and both uncertainties for bulk power reliability evaluation, and it's a black-box and computationally expensive because it's too complicated for us to derive its explicit functional form. From (1), we can see that reliability index Y is actually a random variable affected by both aleatory and epistemic uncertainties. When holding the epistemic variables **U** fixed at a value **u**, i.e. **U** = **u**, the resulting output Y is a function of the aleatory uncertainties **V** solely.