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Probabilistic load flow with correlated input random variables using uniform design sampling

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ABSTRACT

This paper proposes a probabilistic load flow (PLF) methodology using uniform design sampling (UDS). The correlation between input random variables has been taken into consideration. The random numbers of random variables uniformly distributed in (0, 1) are generated by UDS, and subsequently converted into random numbers of input random variables with desired marginal distributions by marginal transformation. Then these random numbers of input random variables are permutated by a method based on rank correlation to satisfy the desired correlation between input random variables. The statistical properties and probability distributions of node voltage and line flow are calculated by Monte Carlo simulation method and statistical method. Considering the uncertainty of correlated wind power and loads, the performance of the proposed PLF methodology is investigated using modified IEEE 14-bus and IEEE 57-bus test systems.

Introduction

Load flow computation is one of the most important tasks for power system operators and planners. Plenty of uncertainty exists in power system operation, such as the variation of loads, forced outage of generators and change of network configurations. The large-scale integration of renewable energy generation and plug-in hybrid electric vehicles has increased the uncertainty. In order to evaluate system load flow more accurately and comprehensively, and provide system operators and planners with more valuable information, uncertainty should be considered in load flow computation. Probabilistic load flow (PLF) is an efficacious technology to investigate the impact of uncertainty on system operation. It can expose the weak points and potential crisis of system operation under various possible uncertainty.

PLF was first proposed by Borkowska [1]. Since then, a number of different methodologies have been proposed to solve the PLF problems, including convolution method [1–3], cumulant method [4–9], point estimate method (PEM) [10–14] and Monte Carlo simulation method (MCSM) [15–24]. The convolution method obtains the probability density function (PDF) of output random variable (node voltage and line flow included) by convolving the PDFs of input random variables involved. This method can deal with the linear correlation between input random variables. Its drawback is time-consuming when considering many input random variables even though fast Fourier transform method [3] is adopted. The cumulant method calculates the probability distribution functions (including PDF and cumulative distribution function (CDF)) of output random variable by arithmetic operation of cumulants and series expansion instead of convolution operation, which comes with a less computational cost. This method needs to linearize the AC nonlinear load flow equations around the operating point. When the variation of input random variable is large, the error due to linearization will be apparent, particularly in the tail region which is furthest from the point of linearization. The PEM obtains an approximate description of the statistical properties of output random variable. This method is computationally efficient if the number of input random variables involved is small. However, the accuracy of this method worsens as the order of the estimated statistical properties becomes higher [12,13]. The computation time needed by PEM and its various versions has either a proportional or an exponential relation in terms of the number of input random variables, which may make them impractical in large-scale power system [20].

MCSM firstly generates random numbers of input random variables according to their marginal distributions, and then does deterministic load flow (DLF) computation for a large number of

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times with inputs of these random numbers to obtain the probability distributions of output random variables. This method can use the exact nonlinear load flow equations and does not need miscellaneous simplifications or complicated mathematical computation. The number of simulations needed to obtain an accurate result by MCSM is independent of system size [9]. Once this method is converged, all the distribution functions of output random variables are simultaneously obtained [7]. So it is considered to be the most accurate, flexible and robust PLF method [18]. Traditional MCSM with simple random sampling (MSRS) can achieve a considerably high accuracy when the sample size is large enough. This method is always adopted as a reference to check the accuracy of other PLF methods [3–20], but its drawback is the need of a large number of simulations.

In order to reduce the computational effort of MCSM and preserve its advantages, a PLF methodology using MCSM with uniform design sampling (MUDS) is proposed in this paper. The uniform design sampling (UDS) technique was first proposed in 1996 [25], which generates random numbers of random variable uniformly distributed in (0,1). Two important enhancements to the UDS proposed in [25] are introduced. The first one is to expand the original UDS to generate random numbers of input random variable with any desired marginal distribution by marginal transformation. The second one is to handle the correlation between input random variables by a method based on rank correlation. Performance of MUDS is investigated using modified IEEE 14-bus and IEEE 57-bus test systems. The uncertainty and correlation of wind power and loads are considered. The results obtained by MUDS are compared with those got by MSRS with regards to both accuracy and execution time criteria.

The remainder of this paper is organized as follows. Section 'Load flow solution' briefly reviews the load flow solution of power system. Next, in Section 'MUDS method', the MUDS method and its computational procedure are analyzed. The performance of MUDS and MSRS is studied in Section 'Case study'. Conclusions are drawn in Section 'Conclusion'.

Load flow solution

Load flow computation is a basic task for the analysis of steady-state as well as dynamic performance of power system. The nonlinear form of AC load flow equations can be expressed as follows:

$$\begin{cases} P_i = V_i \sum_{k=1}^{n} V_k (G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}) \\ Q_i = V_i \sum_{k=1}^{n} V_k (G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik}) \end{cases} \tag{1}$$

$$\begin{cases} P_{ik} = -t_{ik} G_{ik} V_i^2 + V_i V_k (G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}) \\ Q_{ik} = (t_{ik} B_{ik} - b_{ik0}/2) V_i^2 + V_i V_k (G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik}) \end{cases} \tag{2}$$

where P_i and Q_i are the active and reactive power injections at bus i. V_i and V_k are the voltage magnitudes at buses i and k. G_{ik} and B_{ik} are the real and imaginary parts of the admittance matrix of branch ik. θ_{ik} is the difference in voltage angles between buses i and k. P_{ik} and Q_{ik} are the line active and reactive powers in branch ik. t_{ik} is the transformation ratio of branch ik. b_{ik0} is the susceptance of branch ik.

Let $\boldsymbol{W}$ (the boldface symbol denotes a vector or matrix in this paper) be the input vector of active and reactive power injections, $\boldsymbol{X}$ be the state vector of bus voltage magnitudes and angles, $\boldsymbol{Z}$ be the output vector of line active and reactive powers. The nonlinear form of AC load flow equations can be expressed in terms of vector:

$$\begin{cases} \mathbf{X} = f(\mathbf{W}) \\ \mathbf{Z} = g(\mathbf{X}) = g(f(\mathbf{W})) \end{cases} \tag{3}$$

where f and g are the nodal power and line flow functions, respectively.

The DLF uses specific values of input vector $\boldsymbol{W}$ to calculate the specific values of state vector $\boldsymbol{X}$ and output vector $\boldsymbol{Z}$. In the PLF problem, the input vector $\boldsymbol{W}$, state vector $\boldsymbol{X}$ and output vector $\boldsymbol{Z}$ are random variable vectors, which can be described by statistical properties and probability distributions. The convolution method convolves the PDFs of input vector $\boldsymbol{W}$ to obtain the PDFs of state vector $\boldsymbol{X}$ and output vector $\boldsymbol{Z}$. Cumulants of input vector $\boldsymbol{W}$ are adopted in the cumulant method. The cumulants of state vector $\boldsymbol{X}$ and output vector $\boldsymbol{Z}$ are calculated with simple arithmetic process due to properties of cumulants. The first several statistical moments of input vector $\boldsymbol{W}$ are used in PEM. For example, $2m+1$ PEM uses mean, standard deviation, coefficients of skewness and kurtosis to construct three estimated points often named concentrations to represent input vector $\boldsymbol{W}$. MCSM generates N random numbers of input vector $\boldsymbol{W}$ according to its probability distribution and does DLF computation for N times to obtain the N values of state vector $\boldsymbol{X}$ and output vector $\boldsymbol{Z}$. Then the statistical properties and probability distributions of state vector $\boldsymbol{X}$ and output vector $\boldsymbol{Z}$ can be calculated by statistical method.

MUDS method

The procedure of MUDS method can be divided into two main steps, including random numbers generation of input random variables and load flow computation. The basic requirement of random numbers generation of input random variables is to ensure that the generated random numbers preserve both marginal distributions and correlation relationships of input random variables. The values of output random variables can be calculated by doing DLF computation with the random numbers generated above. Then the statistical properties and probability distributions of node voltage and line flow can be obtained by statistical method. Random numbers generation of input random variables mainly contains three steps: (1) Generate random numbers of random variables following uniform distribution on the interval (0,1) by UDS technique; (2) Transform these random numbers to obtain random numbers of input random variables with desired marginal distributions by marginal transformation; (3) Permutate the generated random numbers of input random variables by a method based on rank correlation.

UDS technique

In order to improve the efficiency of experimental design, the uniform design was proposed by Fang and Wang [26,27], which is a type of "space filling" designs. The uniform design seeks to design experimental points uniformly scattered on the domain in a deterministic uniform way. It has been widely applied in manufacturing, system engineering and natural sciences [28,29]. In order to overcome the deficiency of lack of the randomness of statistical analysis, an improvement was proposed, which is the UDS technique. Let N be the sample size of random variable vector $\boldsymbol{Y}_{1\times m} = [y_1,\ldots,y_m]$, which follows uniform distribution on the interval (0,1). The steps of generating random numbers of vector $\boldsymbol{Y}_{1\times m}$ by UDS technique are as follows [25]:

(1) Let Λ be an infinite subset of the positive integer and n belongs to Λ. Generate a positive integer vector $\boldsymbol{H}_{1\times m} = [h_1,\ldots,h_m]$ which satisfies these conditions $h_1 = 1$, $1 < h_j < n$, $h_i \neq h_j$ for all $i \neq j$.
(2) Obtain a vector $\boldsymbol{B}_{1\times m} = [b_1,\ldots,b_m]$ by selecting m independent and identically distributed elements from multinomial distribution $\begin{pmatrix} 0 & 1 & \cdots & N-1 \\ \frac{1}{N} & \frac{1}{N} & \cdots & \frac{1}{N} \end{pmatrix}$.