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An efficient equivalent thermal cost function model for nonlinear mid-term hydrothermal generation planning

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ABSTRACT

This paper presents an efficient model to represent nonlinear thermal generation costs in the mid-term hydrothermal generation planning problem. The proposed approach comprises two procedures: the first one consists in obtaining an exact piecewise quadratic equivalent thermal cost function for total thermal generation cost based on quadratic cost functions for each individual unit. The second procedure consists in using a dynamic piecewise linear model to represent such function in the optimization problem to be solved. The combination of both procedures yield a linearized model for the equivalent thermal generation cost curve for each system area and time step, which are represented in the problem as a set of constraints. Numerical results for some instances of the multi-period, stochastic nonlinear hydrothermal planning problem show a remarkable CPU time reduction and an improved accuracy in the final solution of the problem, as compared to an individual static piecewise linear model, which is the usual approach adopted in the literature.

Introduction

Hydrothermal generation planning is a large-scale, stochastic, nonlinear, multi-stage optimization problem, which is usually solved in a hierarchical way, with long term, medium term and short-term models [1–3]. In particular, mid-term planning usually ranges an horizon from 1 month to 1 year, in weekly or monthly time steps, in a cost-minimization [4–9] or profit-based [10] framework. Uncertainty on hydro inflows, load and/or price may be taken into account, depending on the particular features of each system and the problem considered. In this mid-long term planning horizon, due to the time discretization employed, thermal unit commitment constraints are not considered and thermal generation costs are usually approximated by linear or piecewise linear functions for each unit. As a result, optimization techniques for stochastic linear programming such as Nested Benders decomposition [11], stochastic dual dynamic programming [12], Lagrangian Relaxation [13] or progressive hedging [14] can be employed to solve the problem.

In order to reduce the problem size, some works have proposed to approximate thermal costs as a single function for the aggregate set of thermal plants. In [15] a smooth second order polynomial curve was proposed to approximate the piecewise linear curve for total thermal costs, by using least squares techniques. In [16], a simulation procedure for problems with unit commitment constraints was proposed to derive a curve for total thermal generation as a function of system marginal cost, which was further employed for long-term problems. A nonlinear equivalent thermal curve was also used in [17] for long-term hydrothermal planning but no details are given on how to obtain such curve. Bayón et al. [18] showed that an exact equivalent piecewise quadratic cost function can be obtained based on individual quadratic curves for each unit, and this idea was further extended in [19] to handle more general nonlinear cost functions. Both models were considered for small static hydrothermal optimization problems.

This paper contains two main contributions. First, the piecewise quadratic model proposed in [18] for the equivalent thermal cost curve is extended not only to include lower and upper bounds for generation on each individual unit, but also for its application in the more general mid-term stochastic hydrothermal planning problem. The second contribution of this work is to employ a dynamic piecewise linear (DPWL) model to represent this nonlinear cost curve in the stochastic mid-term hydrothermal planning problem. Numerical results based on the large-scale hydrothermal Brazilian system show a remarkable CPU time reduction as compared to the traditional approach of using static piecewise linear approximation, either for the total system thermal costs or for

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the individual costs of each unit. Moreover, the results obtained by the proposed approach are much more accurate than the usual approaches, since the piecewise linear approximation is performed dynamically as the problem is solved, which allows to employ more accurate discretization near the optimal solution for each node of the stochastic problem.

The paper is organized as follows: In section 'MTHTS problem formulation' we present an overview of the mid-term hydrothermal scheduling problem (MTHTS). In sections 'Piecewise quadratic model for equivalent thermal cost function' and 'Dynamic piecewise linear approximation of the equivalent curve' we describe the two steps of the proposed approach: to construct an equivalent piecewise quadratic curve for total thermal costs and to represent it by a dynamic piecewise linear model in the optimization problem to be solved. Finally, in sections 'Numerical results' and 'Conclusions' we present the numerical results and state the conclusions of this work.

MTHTS problem formulation

The MTHTS problem is a multi-stage stochastic programming problem where the aim is to minimize thermal generation costs, which leads to an optimization of hydro resources in order to dispatch the units in an efficient way regarding use of water. Uncertainties on hydro inflows $I_i^{t,\omega}$ to reservoirs are represented by a scenario tree as shown in Fig. 1. In such tree, we denote as (t, ω) the node related to time step t and scenario ω. The number of scenarios up to time step t is $\Omega(t)$, so that the total number of multistage scenarios is $\Omega(T)$.

Traditional formulation with individual thermal units

The usual formulation of the problem – with individual cost functions for each thermal unit is shown in (1a)–(1d).

$$\min \sum_{t=1}^{T}\left[\sum_{\omega=1}^{\Omega(t)} p^{t,\omega}\sum_{j=1}^{NT}\left(c_{2_j}\left(gt_j^{t,\omega}\right)^2 + c_{1_j} gt_j^{t,\omega} + c_{0j}\right)\right] + \sum_{\omega=1}^{\Omega(T)} p^{t,\omega}\alpha(E^{T,\omega}) \tag{1a}$$

s.t.

$$\sum_{j\in\Pi_s} gt_j^{t,\omega} + \sum_{i\in\Phi_s} gh_i^{t,\omega} + \sum_{l\in\Gamma_s} Int_{l,s}^t = D_s^t \quad \forall\, s,t,\omega \tag{1b}$$

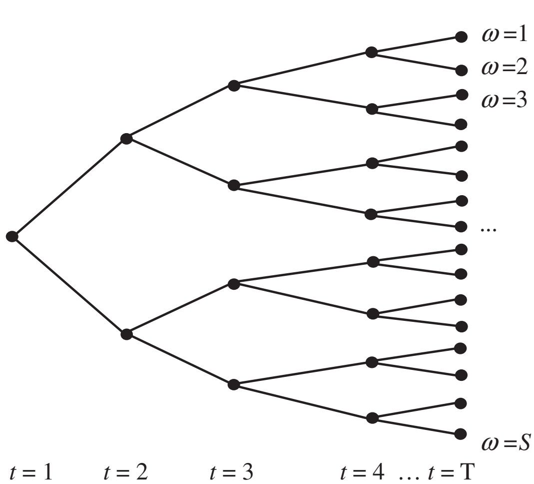

Fig. 1. Representation of uncertainties as a scenario tree.

$$E_i^t + GH_i^t + Sp_i^t = E_i^{t-1} + I_i^t \quad \forall\, i,t, \tag{1c}$$

$$\text{Bounds on } E,\ gh,\ gt,\ Int \tag{1d}$$

The objective function (1a) comprises the sum of total thermal costs throughout all T time steps and corresponding scenarios, each one with a given probability $p^{t,\omega}$. Thermal costs for unit j are given by a second order polynomial function of generation gt, with coefficients c_{2j}, c_{1j} and c_{0j}. At the end of the planning horizon we consider a so called future cost function $\alpha(\cdot)$ in order to reflect future system costs as a function of the vector of end energy storages E in the reservoirs.

The two main constraints of the problem are (1b) and (1c). Eq. (1b) is the energy balance equation: for each system area s, power demand $D_s^{t,\omega}$ should be met by the sum of hydro and thermal generations in the sets Φ_s and Π_s, respectively, of hydro/thermal plants belonging to this area. Possible mismatches are allowed by using available energy interchanges $Int_{l,s}^t$ in neighbourhood systems in the set Γ_s. If necessary, energy shortages can be included as artificial thermal plants with high incremental costs. Eq. (1c) is the energy balance in the reservoirs for each time step/scenario. Since in this paper we are mostly concerned in the representation of the thermal mix, the set of hydro plants are approximated as equivalent energy reservoirs based on the formulations described in [21,22]. Variables $E_i^{t,\omega}$, $gh_i^{t,\omega}$ $Sp_i^{t,\omega}$ denote energy storage, generation and spillage for each equivalent reservoir i. Finally, all variables of the problem have proper lower and upper bounds, as stated in (1d).

Alternative formulation with equivalent thermal plants

In mid and long term problems, network constraints are usually neglected and transmission is represented only by major interchange lines. For this reason, it is not so important where each power plant is located within each system area. As a consequence, thermal generation costs can be considered by a so-called equivalent cost function (ECF), for the total thermal generation in each system area. This is a generalization, to a multi-area setting, of the concepts presented in [18] where a single plant for the whole system was considered.

According to constraints (1b), total thermal generation $GT_S^t = \sum_{j\in\pi_s} gt_j$ in each system area and each time step depends on the vector gh of hydro generation values and Int of interchanges among areas. Moreover, the optimal distribution of this total thermal generation among all thermal units in the area is uniquely defined by the so called coordination equations, which are mathematically formulated as follows (see [23], chapter 3).

$$\begin{cases} \frac{dc_{gt_j}}{dgt_j}(gt_j) = \lambda, & \text{if } \underline{gt_j} < gt_j < \overline{gt_j} \\ \frac{dc_{gt_j}}{dgt_j}(gt_j) \geqslant \lambda, & \text{if } gt_j = \underline{gt_j} \\ \frac{dc_{gt_j}}{dgt_j}(gt_j) \leqslant \lambda, & \text{if } gt_j = \overline{gt_j} \end{cases} \tag{2}$$

where λ is the system marginal cost for the corresponding area (we have suppressed area, time step and scenario indices for simplicity of notation). Constraints (2) correspond to the equal incremental cost property for the units which are not binding at the optimal solution (the so called "marginal units").

Based on condition (2) and on the strongly convex property of the objective function (provided that all second order terms c_2 of the cost functions are greater than zero), each value of total area generation GT_S^t is related to a unique area marginal cost λ, as well as a unique optimal solution $gt_j^*(\lambda)$ for each unit $j \in \Pi_s$. In this sense, the MTHTS can be reformulated by using equivalent thermal plants instead of individual thermal units, as highlighted in expressions (3a)–(3e) below. In this new formulation, each equivalent