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Continuous model in dq frame of Thyristor Controlled Reactors for stability analysis of high power electrical systems

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ABSTRACT

This paper illustrates a formulation of a continuous state space model in dq frame of the Thyristor Controlled Reactor (TCR), which can be used for stability studies by eigenvalue analysis. Differently from the previous approaches, this model is just addressed to reproduce the operation of the non-linear part of the TCR system, without considering shunted capacitor or control system, thus obtaining a characterization of the TCR independent from the application. A typical problem of the continuous models in dq frame of the TCR system is the presence of an un-damped homogenous current component in the solution, which is inconsistent with the real operation; the proposed model solves this problem improving the accuracy. The model performance has been also evaluated in the frequency domain comparing the obtained results with a numerical simulation of the TCR implemented by PSIM program.

Introduction

Thyristor Controlled Reactor (TCR) is a power electronic system, where the equivalent reactance of an inductor at the fundamental frequency is varied by the control of the firing angle of the bidirectional thyristors connected in series with it, thus varying the conduction time of the current flowing in the inductors [1]. This system is often used in Flexible Alternating Current Transmission Systems (FACTS) with the aim to control relevant parameters of the electrical grid. TCR shunted with a capacitor bank is a widespread application and it can be used either in series compensation, for instance the Thyristor Controlled Series Capacitor (TCSC), or in shunt compensation, for example the Static Var Compensator (SVC).

The design and the study of the operation of electrical systems which include TCRs are often based on computer simulations capable of reproducing the instantaneous current and voltage profiles and they provide accurate results. Nevertheless these simulations require a lot of computation time to reproduce the operation of large electrical systems and it is necessary to apply trial and error type studies for the identification of possible critical operating conditions.

On the contrary, analytical models allow a faster approach to the system design [2–4] and are suitable tools for stability analysis (for instance eigenvalue analysis [5–8]).

Different concepts for the model development have been investigated and they may depend on either the applications or the type (continuous [9–14] or discrete time [15–20]), and also other categories have been identified (based on dynamic phasors [21–23]).

With reference to the continuous models in "dq" frame, a typical problem encountered is the presence in the solution of an un-damped homogeneous component at angular frequency $-\omega$, which is inconsistent with the real operation of the TCR system.

A common approach observed in all the referenced models is that the TCR, the shunted capacitor bank and sometime also the control system are considered as a whole; therefore the accuracy is globally evaluated and it can be not so clear if it is due to a good TCR model or to the presence of the other elements that smooth the non-linear and discrete operation of the TCR.

This paper presents an improved TCR continuous model which overcomes the issue related to the un-damped homogeneous component; it is based on a new state space formulation in the dq frame of the TCR system only, without the shunted capacitor bank, thus reproducing the dynamics of the TCR operation more closely. The small signal analysis has been carried out considering the non-linearity of the TCR system. This model can be used for stability studies by eigenvalue analysis; it provides good accuracy in a frequency range wider than the existing continuous models.

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A frequency domain analysis is proposed in the second part of the paper, in order to evaluate the accuracy of the TCR model; it compares the results of the simulations obtained by the TCR state space model (run in Matlab simulink state space tool [24]) with those derived from the numerical simulations of the same system by PSIM program (an electrical transient program) [25].

The TCR continuous model

The study of the TCR system has been carried out assuming a three phases SVC application without shunted capacitor; the block scheme of the TCR in delta connection is shown in Fig. 1, where the voltages and currents are represented in the dq frame.

The relation between the thyristor firing angle α_L and the equivalent star inductance L of the TCR at the fundamental component is [1]:

$$L(\alpha_L) = \frac{L_r}{3 \cdot h(\alpha_L)} \tag{1}$$

where L_r is the inductance of each branch of the TCR and the function $h(\alpha_L)$ is:

$$h(\alpha_L) = \frac{2\pi - 2\alpha_L + \sin(2\alpha_L)}{\pi} \tag{2}$$

The angle α_L is related to the zero cross of the respective phase to phase voltage on the TCR and it may vary from $\pi/2$ rad (maximum consumption of reactive power) to π rad (consumption of reactive power equal to 0). It may be noted that the relation is not linear.

To better appreciate the improvements of the model proposed in this paper and described in Section 'The improved TCR model', the features of a TCR continuous model for SVC applications often proposed in the literature [9–12] are discussed first. At this purpose, a block scheme is shown in Fig. 2, which represents the transfer function of this model. The voltage and current variables are given in direct-quadrature frame obtained by the dq transformation applied to the three phase system abc (the zero component is neglected) and reported below for a generic variable x:

$$\begin{bmatrix} x_d(t) \\ x_q(t) \end{bmatrix} = \frac{2}{3} \cdot \begin{bmatrix} \cos(\omega \cdot t + \theta_r) & \cos(\omega \cdot t + \theta_r - \frac{2\pi}{3}) & \cos\left(\omega \cdot t + \theta_r - \frac{4\pi}{3}\right) \\ -\sin(\omega \cdot t + \theta_r) & -\sin\left(\omega \cdot t + \theta_r - \frac{2\pi}{3}\right) & -\sin\left(\omega \cdot t + \theta_r - \frac{4\pi}{3}\right) \end{bmatrix} \cdot \begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix} \tag{3}$$

where ω is the angular frequency at the fundamental component of the grid (in this paper the frequency of the grid is assumed at 50 Hz and ω is equal to 314 rad/s) and θ_r is the reference phase angle of the dq frame. In Appendix the definitions of forward and backward sequences in a dq frame are quoted.

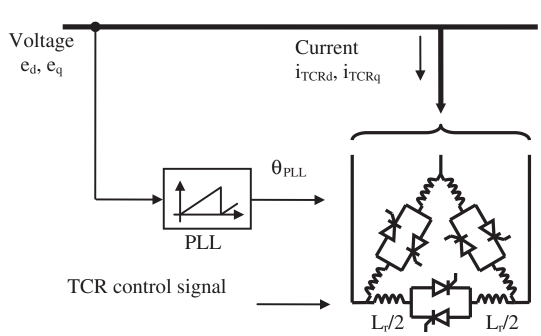

Fig. 1. The block scheme of the TCR system with the variables given in dq frame.

The inputs of the model are the firing angle α_{TCR} of the thyristors, the synchronization angle θ_{PLL} of the Phase Locked Loop (PLL) and the dq components of the busbar voltages e_d and e_q, while the output variables are the TCR currents i_{TCRd} and i_{TCRq}. The block scheme has been divided in two parts (see the two boxes in Fig. 2), where the interface signals are ed_L and eq_L, to facilitate the explanation of the improved model features described in the next section.

A PLL has been assumed to synchronize the TCR to the ac input voltages [26]; the effects of the PLL are accounted by the output angle θ_{PLL} and the variation of the voltage phase angle $\theta_V = \arctan(e_q/e_d)$, as shown in Fig. 2. It is highlighted that the θ_{PLL} signal considered here does not include the sawtooth component, but only the average one in a period, necessary for the synchronization with the voltage e_d and e_q; this means that in steady state and if e_d and e_q are constant, the signal θ_{PLL} is equal to θ_V.

Two transfer functions $G_1(s)$ and $G_2(s)$ are used (Fig. 2) to describe the dynamics of the TCR system [9–13]:

$$\begin{aligned} G_1(s) &= e^{-sTd} \\ G_2(s) &= \tfrac{1}{1 + sT_b} \end{aligned} \tag{4}$$

where $G_1(s)$ is related to the time delay T_d of firing angle assumed as the average time between two switching instants (of about one ms) and $G_2(s)$ describes the TCR switching operation accounting for the three phases arrangement and it is represented by a low pass filter with time constant T_b (value between 3 and 6 ms), depending on the conduction angle of the thyristor of the TCR.

The dq components of the TCR currents (i_{TCRd} and i_{TCRq}) are calculated by the system DE of differential equations:

$$\begin{bmatrix} \frac{di_{TCRd}}{dt} \\ \frac{di_{TCRq}}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix} \cdot \begin{bmatrix} i_{TCRd} \\ i_{TCRq} \end{bmatrix} + \begin{bmatrix} inp_{DEd} \\ inp_{DEq} \end{bmatrix} \tag{5}$$

where inp_{DEd} and inp_{DEq} are the input dq components of the system DE; they depend on the firing angle of the thyristor α_{TCR}, on the output angle θ_{PLL} of the PLL and on the voltages e_d and e_q; moreover, the two transfer functions $G_1(s)$ and $G_2(s)$ of Fig. 2 are the key elements to approximate the TCR discrete operation.

This model offers a good representation of the TCR dynamic apart from the presence of a homogeneous component in the TCR dq currents not consistent with the real TCR operation. The presence of this component was already observed in the dq voltages of the TCSC continuous models in [17,18] (in this case an equivalent capacitor is considered, thus resulting in a duality current–voltage with the SVC model). A mathematical justification of the homogeneous component is given below.

The homogenous component in the TCR currents

The block DE of the TCR model accounts for the dq equations (5) which represent the behavior of an ideal three-phase inductor. In equivalent star configuration in the abc frame, those equations can be written as: