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Unit commitment using Lagrangian relaxation and particle swarm optimization

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ABSTRACT

Unit commitment (UC) is a NP-hard nonlinear mixed-integer optimization problem. This paper proposes ELRPSO, an algorithm to solve the UC problem using Lagrangian relaxation (LR) and particle swarm optimization (PSO). ELRPSO employs a state-of-the-art powerful PSO variant called comprehensive learning PSO to find a feasible near-optimal UC schedule. Each particle represents Lagrangian multipliers. The PSO uses a low level LR procedure, a reserve repairing heuristic, a unit decommitment heuristic, and an economic dispatch heuristic to obtain a feasible UC schedule for each particle. The reserve repairing heuristic addresses the spinning reserve and minimum up/down time constraints simultaneously. Moreover, the reserve repairing and unit decommitment heuristics consider committing/decommitting a unit for a consecutive period of hours at a time in order to reduce the total startup cost. Each particle is initialized using the Lagrangian multipliers obtained from a LR that iteratively updates the multipliers through an adaptive subgradient heuristic, because the multipliers obtained from the LR tend to be close to the optimal multipliers and have a high potential to lead to a feasible near-optimal UC schedule. Numerical results on test thermal power systems of 10, 20, 40, 60, 80, and 100 units demonstrate that ELRPSO is able to find a low-cost UC schedule in a short time and is robust in performance.

Introduction

Unit commitment (UC) [1] refers to scheduling on/off status and power outputs of a power system's generating units over a short-term planning horizon (e.g. 1 day), with the objective of minimizing total power generation cost while simultaneously satisfying coupling constraints of power balance and spinning reserve, as well as physical and operational constraints of each individual unit. UC is critical to the daily planning of modern power systems, as an improved UC schedule may significantly save the power generation cost, e.g. by millions of dollars per year [2].

UC has commonly been formulated as a nonlinear mixed-integer optimization problem, which is NP-hard [3]. Exhaustive enumeration is certainly able to find an exact optimal UC schedule, but it is inapplicable to a realistic power system (which usually comprises tens to hundreds of units) due to its prohibitive exponential computation time requirement. Accordingly, existing research endeavors have mainly focused on deriving a near-optimal UC schedule using various optimization algorithms. Extensive literature surveys on the UC problem can be found in [4,5].

Lagrangian relaxation (LR) [2,6–15] and particle swarm optimization (PSO) [10,12,16–20] are two popular optimization algorithms that have been applied to solve the UC problem. LR relaxes the complicating constraints of power balance and spinning reserve with the introduction of Lagrangian multipliers, resulting in a dual problem that is much easier to solve. LR iteratively updates the multipliers and solves the associated dual problem. PSO is a modern meta-heuristic optimization algorithm introduced in 1995 [21,22]. PSO simulates the movements of organisms in a bird flock or fish school, as it is inspired by the natural process of group communication to share individual knowledge. PSO is population based and finds the optimum using a swarm of particles, with each particle representing a candidate solution, thus PSO is strong in parallel global search. Compared with evolutionary computation based meta-heuristics such as evolutionary programming (EP) [23] and genetic algorithm (GA) [9,24–27], PSO basically does not use any evolution operator (e.g. crossover, mutation, or selection), thus PSO is simpler in concept and easier to implement. When applying PSO to UC, there are three particle representation schemes: one is LR based PSO using Lagrangian multipliers to encode a particle [10,12], as a UC schedule can be obtained through solving a Lagrangian dual problem; the second is binary PSO using an on/off status schedule [17–20],

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as the power outputs can be obtained via economic dispatch (ED) [1]; and the last is binary relaxation based PSO using real numbers in [0, 1] to approximate an on/off status schedule [16]. For LR [2,6–15] (including LRGA [9] and the LR based PSOs [10,12]), it is usually time consuming to find a feasible near-optimal UC schedule, because small changes in the Lagrangian multipliers can cause over-correction of the on/off status of the units, leading to violation of the relaxed spinning reserve constraints. In the binary and binary relaxation based PSOs [16–20], the particles initialized and updated are also often infeasible with respect to the spinning reserve constraints. As a consequence, many literature works [2,8,11,13,14,17–20] proposed to make a UC schedule reserve-feasible through successively committing an uncommitted unit and/or further refine a reserve-feasible schedule through successively decommitting an over-committed unit. The unit for commitment/decommitment is usually selected according to its full load average cost (FLAC).

This paper proposes ELRPSO, an enhanced method that combines the strengths of LR and PSO to solve UC. ELRPSO employs a state-of-the-art powerful PSO variant called comprehensive learning PSO (CLPSO) [28] to find a feasible near-optimal UC schedule. In PSO, all particles "fly" in the search space. PSO relies on iterative learning to find the optimum. In each iteration (or generation), a particle adjusts its position based on its personal search experience and also the experiences of its neighborhood particles. With respect to many existing PSO variants such as global PSO [29], local PSO [30], and fully informed PSO [31], once a neighborhood experiences related exemplar position has been determined, it is used to update a particle's position on all dimensions. Recognizing the fact that one exemplar does not always offer a good guide on every dimension, CLPSO encourages a particle to learn from different exemplars on different dimensions, thus CLPSO performs excellent in preserving the particle's diversity and locating the global optimum region. In ELRPSO, a priority list based on ascending order of FLACs of the units is first set up. Each particle in the employed CLPSO represents Lagrangian multipliers. In each generation, the CLPSO uses a low level LR procedure to find an on/off status schedule for each particle according to the Lagrangian multipliers that the particle represents, a reserve repairing heuristic to make the schedule reserve-feasible, a unit decommitment heuristic to handle over-commitment(s) in the schedule, and an ED heuristic to determine the power outputs of the units so as not to violate the power balance constraints. Both the reserve repairing and unit decommitment heuristics rely on the FLACs sorted in the priority list to successively select a unit for commitment/decommitment. Each particle in the CLPSO is initialized using the Lagrangian multipliers obtained from a separate run of a LR. Given random initial Lagrangian multipliers, the LR uses an adaptive subgradient heuristic to iteratively update the multipliers. As the Lagrangian function formed by the relaxation of the complicating constraints is concave and non-differentiable, the subgradient heuristic can make the Lagrangian multipliers move closer to the optimal multipliers in each iteration through the use of appropriate step sizes in the direction of subgradients [32]. After an enough number of iterations, the Lagrangian multipliers obtained are expected to be near the optimal multipliers, thus the on/off status schedule found from the low level LR procedure in the CLPSO using the initial particle obtained from the LR is usually similar to the dual optimal schedule, only with a small number of different on/off commitments in the two schedules. Our observation from the numerical results as presented in section 'Simulation and Numerical Results' is that such an on/off status schedule attained from the low level LR procedure has a high potential to lead to a feasible near-optimal schedule through the use of the reserve repairing and unit decommitment heuristics in the CLPSO, whereas in contrast, an on/off status schedule that is much different from the dual optimum often leads to a feasible sub-optimal schedule using the heuristics.

ELRPSO is able to find a feasible near-optimal UC schedule in a short time for three reasons: (1) the LR uses a subgradient heuristic that is adaptive in determining the step sizes for the subgradients, thus the Lagrangian multipliers can quickly move sufficiently close to the optimal multipliers; (2) the CLPSO obtains a feasible UC schedule from each particle in each generation, thus the CLPSO searches exclusively in the feasible space; and (3) the CLPSO searches in a space near the optimal Lagrangian multipliers, and small changes in the multipliers can cause over-correction of the on/off status of the units, thus a feasible near-optimal UC schedule is highly likely to be found from a variety of on/off status schedules (similar to the dual optimum) explored in a small number of function evaluations.

ELRPSO consumes considerably less memory storage than the binary and binary relaxation based PSOs proposed in [16–20]. The particle representation in ELRPSO is independent of system size. Besides, without explicit parallel programming, the particles are actually handled one by one in each generation on a computer, thus storage for storing the on/off status and power outputs can be shared among all the particles. ELRPSO differs from the LR based PSOs proposed in [10,12] in that ELRPSO uses LR to help initialize the particles in a space near the optimal Lagrangian multipliers and a heuristic to make the schedules reserve-feasible.

The reserve repairing and unit decommitment heuristics proposed in this paper are novel. When committing an uncommitted unit, the reserve repairing heuristic also modifies some relevant on/off commitments so as not to violate the minimum up/down time constraints; or in other words, it addresses the spinning reserve and minimum up/down time constraints simultaneously, whereas existing literature works either ignore the latter constraints [2,8] or handle the two types of constraints separately (and is hence less efficient) [17–20]. Different from [2,8,11,13,14,17–20] that commit/decommit a unit for 1 h at a time, the reserve repairing and unit decommitment heuristics consider committing/decommitting a unit for a consecutive period of hours at a time in order to reduce the frequency of starting the unit and thus reduce the total startup cost.

The rest of this paper is organized as follows. In section 'Problem Formulation and Related Works', a mathematical formulation of the UC problem is given and literature works related to UC are discussed. Section 'Methodologies' reviews LR and CLPSO. Section 'Algorithm Framework of ELRPSO' states the general framework of ELRPSO. Details of the LR and CLPSO implemented in ELRPSO are elaborated in sections 'Implementation of Lagrangian Relaxation in ELRPSO' and 'Implementation of Comprehensive Learning Particle Swarm Optimization in ELRPSO' respectively. In section 'Simulation and Numerical Results', numerical results are presented. Section 'Conclusions' concludes the paper.

Problem formulation and related works

Problem formulation

Before putting the mathematical formulation of the UC problem into perspective, decision variables and model parameters are defined as follows.

Decision variables:

$P_{i,t}$	Power output of unit i at hour t, in MW
$U_{i,t}$	On/off status of unit i at hour t (on = 1, off = 0)