Electrical Power and Energy Systems 58 (2014) 242–245

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Short Communication

An affine arithmetic-based algorithm for radial distribution system power flow with uncertainties

Wei Gu [a,*], Lizi Luo [a], Tao Ding [b], Xiaoli Meng [c], Wanxing Sheng [c]

[a] School of Electrical Engineering, Southeast University, Nanjing 210096, China
[b] Department of Electrical Engineering, Tsinghua University, Beijing 100084, China
[c] China Electric Power Research Institute, Beijing 100192, China

ARTICLE INFO

Article history:
Received 15 June 2013
Received in revised form 30 December 2013
Accepted 16 January 2014

Keywords:
Affine arithmetic
Interval arithmetic
Radial distribution system power flow
Uncertainties

ABSTRACT

This letter presents an algorithm for radial distribution system power flow in the presence of uncertainties. To reduce the overestimation of bounds yielded by correlation of variables in interval arithmetic (IA), affine arithmetic (AA) is applied in this study to carry out tests of distribution system power flow. Compared with the algorithm based on IA, the proposed algorithm narrows the gap between the upper and lower bounds of the power flow solution. IEEE 33-bus and 69-bus test systems are used to demonstrate the effectiveness of the proposed algorithm.

1. Introduction

With an increasing number of renewable power generation systems connected to distribution systems, power injections are increasingly difficult to model, which makes it difficult for traditional deterministic methods to calculate the power flow. To deal with the uncertainties in power systems, many modeling approaches have been proposed in Refs. [1–5]. Moreover, popular power flow algorithms, such as Newton-Raphson and Fast Decoupled algorithms, generally fail to converge when analyzing the distribution system for its radial structure and high R/X ratio [6]. Therefore, the backward/forward sweep algorithm is used to solve this problem. In Ref. [7], an IA-based backward/forward sweep algorithm is developed to calculate the power flow of radial distribution systems, which uses ranges restricted by upper and lower bounds to express the uncertainties.

However, the ranges estimated by IA tend to be too large, especially in complicated expressions or long iterative computations, because they ignore the correlation of different variables [8]. An AA-based algorithm is proposed to reduce this overestimation of bounds. This algorithm uses the affine form instead of the interval form to describe the uncertainties of power injections, thus accounting for correlations between different variables.

2. Affine arithmetic-based algorithm for radial distribution system power flow

2.1. Concepts of affine arithmetic

In affine arithmetic, a quantity x is represented by an expression of the form

$$\hat{x} = x_0 + x_1\varepsilon_1 + \cdots + x_n\varepsilon_n \tag{1}$$

which is an affine expression of noise symbols ε_i with real coefficients x_i. Each noise symbol ε_i is a symbolic real variable whose value is unknown except that it is restricted to the interval $[-1, +1]$ and is independent from other noise symbols. The coefficient x_0 is called the central value of the affine form of $\hat{x}$. The coefficients $x_1, \ldots, x_n$ are the partial deviations associated with the noise symbols $\varepsilon_1, \ldots, \varepsilon_n$ in $\hat{x}$. The number n of noise symbols depends on the affine form. Different affine forms use a different number of noise symbols, some of which may be shared with other affine forms.

Affine forms provide interval bounds for the corresponding quantities: If a quantity x is represented with the affine form $\hat{x}$ as above, then $x \in [x_0 - r_x, x_0 + r_x]$. Here $r_x = |x_1| + \ldots + |x_n|$ is called the total deviation of $\hat{x}$ [9].

2.2. Calculation rules of affine arithmetic applied in complex field

Two types of uncertainty sources exist in the power injections: active and reactive power. Because reactive power is represented

* Corresponding author. Tel.: +86 13814005169; fax: +86 25 87796196.
E-mail address: wgu@seu.edu.cn (W. Gu).

http://dx.doi.org/10.1016/j.ijepes.2014.01.025
0142-0615/© 2014 Elsevier Ltd. All rights reserved.

by an imaginary number, complex affine forms are used for the analysis of power flow with these uncertainties.

Consider two complex quantities x and y represented by the complex affine forms

$$\begin{aligned}\hat{x} &= x_0 + x_1\varepsilon_1 + \cdots + x_{2n}\varepsilon_{2n}\\ \hat{y} &= y_0 + y_1\varepsilon_1 + \cdots + y_{2n}\varepsilon_{2n}\end{aligned} \tag{2}$$

where x_0 and y_0 are complex numbers and represent the central value of each complex affine form. When i is odd, x_i and y_i are real coefficients that represent the uncertainties derived from the active power injections. When i is even, x_i and y_i are imaginary coefficients that represent the uncertainties derived from the reactive power injections.

Based on the calculation rules of affine arithmetic applied in real number field described in Ref. [9], addition, subtraction, multiplication and division of these two complex affine forms can be derived as follows:

$$\hat{x} + \hat{y} = (x_0 + y_0) + (x_1 + y_1)\varepsilon_1 + \cdots + (x_{2n} + y_{2n})\varepsilon_{2n} \tag{3}$$

$$\hat{x} - \hat{y} = (x_0 - y_0) + (x_1 - y_1)\varepsilon_1 + \cdots + (x_{2n} - y_{2n})\varepsilon_{2n} \tag{4}$$

$$\begin{aligned}\hat{x}\cdot\hat{y} &= \left(x_0 + \sum_{i=1}^{2n} x_i\varepsilon_i\right)\left(y_0 + \sum_{i=1}^{2n} y_i\varepsilon_i\right)\\ &= x_0y_0 + \sum_{i=1}^{2n}(x_0y_i + y_0x_i)\varepsilon_i + \left[\sum_{i=1}^{2n} f(x_i)\right]\cdot\left[\sum_{i=1}^{2n} f(y_i)\right]\varepsilon_{2n+1}\end{aligned} \tag{5}$$

where ε_{2n+1} is a new noise symbol that is created during the computation. For any complex number $z = a + jb$, the function $f()$ is defined as $f(z) = |a| + j|b|$.

$$\begin{aligned}\frac{\hat{x}}{\hat{y}} &= x_0\cdot C + \sum_{i=1}^{2n}\left(C\cdot x_i - \frac{1}{AB}\cdot x_0y_i\right)\varepsilon_i + D\cdot x_0\cdot\varepsilon_{2n+1} + \left[\sum_{i=1}^{2n} f(x_i)\right]\\ &\cdot\left[\sum_{i=1}^{2n} f\left(-\frac{1}{AB}\cdot y_i\right) + f(D)\right]\varepsilon_{2n+2}\end{aligned} \tag{6}$$

where ε_{2n+1}, ε_{2n+2} are new noise symbols that are created during the computation. These new variables are defined as $A = y_0 - \sum_{i=1}^{2n} f(y_i)$, $B = y_0 + \sum_{i=1}^{2n} f(y_i)$, $C = \frac{B+A+2\sqrt{AB}}{2AB} - \frac{1}{AB}y_0$ and $D = \frac{B+A-2\sqrt{AB}}{2AB}$, and the function $f()$ has the same definition as in multiplication.

2.3. Steps to radial distribution system power flow analysis

The basic power flow analysis method used in this study is the backward/forward sweep power flow algorithm. However, to represent the uncertainties of the active and reactive power, power injections of each node have been treated as affine forms rather than fixed numbers, and consequently, the complex arithmetic has been replaced by complex affine arithmetic.

Concrete steps are as follows:

Step 1: Number the nodes of the distribution system and define the node count as N.

Step 2: Transform the given power injections into affine forms with $2N$ noise symbols, with each node corresponding to two noise symbols. For example, assuming the power injection of node m is $P + jQ$ and has a $\pm k\%$ tolerance, it can be represented by

$$\hat{S}_m = P + jQ + (P\cdot\varepsilon_{2m-1} + jQ\cdot\varepsilon_{2m})\cdot k\%. \tag{7}$$

The irrelevant noise symbols $\varepsilon_1, \ldots, \varepsilon_{2m-2}, \varepsilon_{2m+1}, \ldots, \varepsilon_{2N}$ cannot be found in the formula because the coefficients associated with them just equal to zero.

Step 3: Obtain the given voltage at the root node and set the initial voltages to all the other nodes as 1 p.u.

With the preceding work above, the following steps are implemented for the iterative solution of the system.

Step 4: At iteration k, the nodal current injection $I_i^{(k)}$ at node i can be calculated by

$$I_i^{(k)} = \left(S_i/U_i^{(k-1)}\right)^* \tag{8}$$

where S_i is the power injection at node i expressed by affine forms in Step 2 and $U_i^{(k-1)}$ is the calculated voltage at node i during the $(k-1)$th iteration.

Step 5: The details of this step are the same as the traditional backward/forward sweep except that the complex arithmetic has been replaced by complex affine arithmetic. At iteration k, the nodal affine form voltage $U_i^{(k)}$ is obtained by this step.

Step 6: At the end of iteration k, the distance between $U_i^{(k)}$ and $U_i^{(k-1)}$, henceforth denoted by d_i, needs to be calculated for all nodes i. Consider an affine form voltage expressed as

$$\hat{U} = u_0 + u_1\varepsilon_1 + \cdots + u_n\varepsilon_n \tag{9}$$

its corresponding interval form with upper and lower bounds can be represented by

$$[\underline{U},\ \overline{U}] = [u_0 - r_U,\ u_0 + r_U] = \left[u_0 - \sum_{i=1}^{n} f(u_i),\ u_0 + \sum_{i=1}^{n} f(u_i)\right] \tag{10}$$

where r_U is the total deviation of $\hat{U}$ and $f()$ has the same definition as formulation (5)–(6). After transforming all the affine form voltages into interval forms according to (10), d_i is calculated by

$$d_i = \max\left(\left|\underline{U}_i^{(k)} - \underline{U}_i^{(k-1)}\right|,\ \left|\overline{U}_i^{(k)} - \overline{U}_i^{(k-1)}\right|\right) \tag{11}$$

If $\max(d_i)$, $i = 1, 2, \ldots, N$, is less than the specified voltage error tolerance limit, the power flow analysis has converged. Otherwise the algorithm goes back to Step 4 to proceed with next iteration.

3. Case studies

To demonstrate the effectiveness of the proposed algorithm, it is implemented on IEEE 33-bus and 69-bus test systems. In this letter, the voltage error tolerance limit for convergence of the iterative process is 10^{-4} p.u.

First, the proposed algorithm is used to analyze the 33-bus system with an assumed ±20% tolerance on the given power injection of each bus. The power flow solutions obtained by different algorithms are compared in Figs. 1 and 2, with Fig. 1 showing the bus voltage magnitude bounds and Fig. 2 depicting the bus voltage angle bounds. Algorithm A is the algorithm proposed by this letter, algorithm B represents the interval arithmetic-based algorithm [7] and algorithm C is the Monte Carlo method with 10,000 trials. Obviously, the ranges of solutions obtained by both the AA-based and IA-based algorithms completely contain the bounds obtained by the Monte Carlo method. This overlap demonstrates that both of the first two algorithms can give proper approximations of the power flow solution bounds. Notice also that the solution ranges of the AA-based algorithm are narrower than that of IA-based algorithm, which proves that the AA-based algorithm is able to reduce the overestimation of bounds compared with the IA-based algorithm, due to its accounting for correlations between different variables in the distribution system.

Next, both the 33-bus and the 69-bus test systems with uncertainty tolerances from ±10% to ±50% are analyzed. The maximum errors of bus voltage magnitudes and angles under different