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# Existence and uniqueness of conjectured supply function equilibria

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ABSTRACT

Supply Function Equilibrium (SFE) and Conjectured Supply Function Equilibrium (CSFE) are some of the approaches most used to model electricity markets in the medium and long term.

SFE represents the generators' strategies with functions that link prices and quantities, but leads to systems of differential equations hard to solve, unless linearity is assumed (Linear Supply Function Equilibrium, LSFE). CSFE also assumes linearity of the supply functions but only around the equilibrium point, also avoiding the system of differential equations.

This paper analyzes the existence and uniqueness of G-CSFE (a CSFE previously proposed by the Authors) for both elastic and inelastic demands. In addition, it also proves that the iterative algorithm proposed to compute G-CSFE has a fixed point structure and is convergent, and that LSFE is a particular case of G-CSFE when demand and marginal costs are linear. Selected examples show the performance of G-CSFE and how it can be applied to market power analysis with meaningful results.



## 1. Introduction

Several approaches have been proposed in the literature to compute and analyze the behavior of the generators in oligopolistic electricity markets (see [1,2]). Although most of these models have in common the simultaneous profit maximization of the generators (Nash equilibrium approach, see [3,4]), they differ in how the strategies of the generators are modeled. For example, strategies can be prices (Bertrand competition, see [5]), quantities (Cournot and Conjectural Variation approaches, see [6–9]), or supply functions, relating prices and quantities (supply function [10–12] and conjectured supply function approaches [2,13,14]).

Conjectured Supply Function Equilibrium (CSFE) is a combination of the conjectural variation and the supply function approaches, where the generators' strategies are represented by first order local approximations around the equilibrium point [2]. For each generator its conjecture is the slope of its residual demand curve (conjecture price-response, see [15,16]),[1] which, at the equilibrium point, is calculated from the slopes of the first order approximations of its competitors bidding curves (see [2,15,17]). Conjectures internalize most of the strategic behavior of the generators by representing their influence on the market price [16], but their computation depends on the CSFE approach selected, since the parameters of the linear approximations are sometimes supposed partially known (slope known and intercept unknown or vice versa), or totally unknown [2].

Knowing the slopes is equivalent to knowing the conjectures, since the latter can be directly computed from the former (see [14,15]). Very often these conjectures are estimated from historic generator bidding curves (see [7,15,18]) and kept constant for all the forecasting horizon. However its use for medium and long term in a dynamic environment is rather arguable since conjectures should vary to reflect how the generators' strategies adapt to the evolving market structure and regulation (see [16,19]).

When the intercepts are given and the slopes are variables, the conjectures result from the equilibrium (see [2]) but are strongly conditioned by the given intercepts [16]. Therefore similar remarks as above about its applicability to medium and long term can be made.

When both intercepts and slopes are decision variables (G-CSFE), the supply functions of the generators are computed according to the market structure, and so are the conjectures. Thus, the generators strategies are dynamic and adapt to the market evolution, being more suited for medium and long term studies. However additional hypothesis must be considered to get a well-defined problem [16]. Ref. [13] presents an efficient formulation for the G-CSFE and proposes an iterative algorithm to solve it. In addition, unlike the Linear Supply Function Equilibrium (LSFE, see [11,20,21]), if the G-CSFE is solved for different demand values, a complete supply function, not necessarily linear, can be obtained for each generator (see [2,9]). However existence and uniqueness of the G-CSFE has not yet been

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[1] Strictly talking, conjectures represent any assumption about the behaviour of the competitors [16]. However, in this specific case we assume that the conjecture, for a generator, corresponds to the slope of its residual demand slope (conjecture price-response) which, at the equilibrium, is consistent with the slopes of the supply function of its competitors [15].

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analyzed. While the elastic case, under the assumption that both the marginal costs and the system demand are linear functions, can be considered already studied in [22], this article proves that this result cannot be directly applied to the inelastic demand case. On the contrary, specific considerations must be made to guarantee the existence and uniqueness of the solution.

It is important to emphasize that perfectly inelastic demand assumption for medium and long term analysis is a very common and sensible approach for avoiding the estimation of the demand elasticity in scenarios, see [23], where the future demand does not change as much as the market price does (see [24,9]).

This article has three main objectives. It proves that the G-CSFE includes, as a particular case, the LSFE when both the marginal costs and the elastic system demand are linear functions. Unlike [25] where only the symmetric case was analyzed, it also determines the conditions for the existence and uniqueness of the G-CSFE for the asymmetric case with both elastic and inelastic demands, proving that the inelastic case is not a particular case of the elastic one. Finally it determines the mathematical conditions under which the iterative algorithm proposed in [13] to compute the G-CSFE converges to the Nash-equilibrium, and presents some illustrative examples that show the applicability of the proposed algorithm to the computation of the market power of the generators (see [26] for a review on market power indices).

The article is organized as follows. Section 2 presents the formulation of the LSFE approach outlined in [11]. Section 3 describes the G-CSFE proposed in [13] for both elastic and inelastic demand representation, and proves that, when the demand and the marginal costs are linear, the LSFE is a particular case of the G-CSFE. Section 4 proves the existence and uniqueness of the G-CSFE when the marginal costs are supposed to be linear-increasing. Section 5 tries to generalize the above result by extending it to the case of nonlinear-increasing marginal costs showing that in this case is not possible to guarantee the existence and uniqueness in a general way. Section 6 presents the necessary conditions under which the iterative algorithm proposed in [13] to compute the G-CSFE reaches the Nash-equilibrium. Section 7 discusses some illustrative examples about the equilibrium's existence showing the strong relationship between the conjectures and the structure of the market. Finally, Section 8 presents the main conclusions.

## 2. Linear supply function equilibrium

In the LSFE the production of the $i$th generator $P_i$ is linearly related with its offer price $\pi_i$ in the following way:

$$P_i(\pi_i) = \alpha_i \cdot (\pi_i - \gamma_i) \quad \forall i \in E \tag{1}$$

$E$ being the set of generators, $\alpha_i \geqslant 0$ the slope of the function ($\alpha_i = \partial P_i / \partial \pi_i$) and $\gamma_i$ the minimum offer price ($\pi_i \geqslant \gamma_i$), both being decision variables.

The generation's costs are supposed to be quadratic, that is:

$$C_i(P_i(\pi_i)) = \frac{1}{2} \cdot c_i \cdot (P_i(\pi_i))^2 + b_i \cdot P_i(\pi_i) + a_i \quad \forall i \in E \tag{2}$$

$c_i$, $b_i$ and $a_i$ being the fixed coefficients of the costs' functions, leading to linear marginal costs.

The balance between the total production and the elastic system demand $D(\pi)$ is given by:

$$D(\lambda) = \sum_{i \in E} P_i(\lambda) \tag{3}$$

where $\lambda$ is the equilibrium market price and $\partial D(\pi)/\partial \pi = -\alpha^0$, $\alpha^0$ being the demand's elasticity[2] and $\pi$ the price that consumers are willing to pay for a certain amount $D$.

[2] Strictly speaking the demand elasticity is $(\Delta D/D)/(\Delta \lambda/\lambda) = -(\lambda/D) \cdot \alpha^0$. However, here for simplicity the elasticity refers to the slope $\alpha^0$.

The profit $B_i(\lambda)$ for each generator $i$ is given by:

$$B_i(\lambda) = \lambda \cdot P_i(\lambda) - C_i(P_i(\lambda)) \quad \forall i \in E \tag{4}$$

where $\lambda \cdot P_i(\lambda)$ represents the incomes and $C_i(P_i(\lambda))$ the costs.

First order equilibrium conditions are obtained by deriving (4) with respect to the price $\lambda$ (decision variable for the LSFE, see [11]) and setting equal to zero, that is:

$$\begin{aligned} &\frac{\partial B_i(\lambda)}{\partial \lambda} = P_i(\lambda) + \lambda \cdot \frac{\partial P_i(\lambda)}{\partial \lambda} - \frac{\partial C_i(P_i(\lambda))}{\partial P_i} \cdot \frac{\partial P_i(\lambda)}{\partial \lambda} = 0 \\ &\Rightarrow P_i(\lambda) + \frac{\partial P_i(\lambda)}{\partial \lambda} \cdot \left(\lambda - \frac{\partial C_i(P_i(\lambda))}{\partial P_i}\right) = 0 \quad \forall i \in E \end{aligned} \tag{5}$$

where the derivative of the generator production with respect to the market price is the slope of its supply function, that is:

$$\begin{aligned} &P_i(\pi) = D(\pi) - \sum_{j \neq i} P_j(\pi) \\ &\Rightarrow \left.\frac{\partial P_i(\pi)}{\partial \pi}\right|_{\pi=\lambda} = \frac{\partial P_i(\lambda)}{\partial \lambda} = -\alpha^0 - \sum_{j \neq i} \alpha_j \end{aligned} \tag{6}$$

Substituting (1), (2), and (6), into (5) leads to:

$$\alpha_i \cdot (\lambda - \gamma_i) = (\alpha^0 + \sum_{j \neq i} \alpha_j) \cdot (\lambda - (c_i \cdot (\alpha_i \cdot (\lambda - \gamma_i)) + b_i)) \quad \forall i \in E \tag{7}$$

In the LSFE the supply functions of the generators are assumed to be consistent across all times (see [20]), which implies that (7) must be valid for any realization of $\lambda$. Equating coefficients of $\lambda$:

$$\alpha_i = (\alpha^0 + \sum_{j \neq i} \alpha_j) \cdot (1 - c_i \cdot \alpha_i) \quad \forall i \in E \tag{8}$$

Equating coefficients of the constant term:

$$-\alpha_i \cdot \gamma_i = (\alpha^0 + \sum_{j \neq i} \alpha_j) \cdot (c_i \cdot \alpha_i \cdot \gamma_i - b_i) \quad \forall i \in E \tag{9}$$

Substituting the right side of (8) only into the left side of (9) and simplifying yields (see [11] for more details):

$$\gamma_i = b_i \quad \forall i \in E \tag{10}$$

Eq. (10) means that the minimum offer price of each generator is its minimum marginal costs. Dividing (9) by $-\gamma_i$ and clearing $\alpha_i$, with $b_i = \gamma_i$, leads to:

$$\alpha_i = \frac{\alpha^0 + \sum_{j \neq i} \alpha_j}{1 + c_i \cdot \left(\alpha^0 + \sum_{j \in E}^{j \neq i} \alpha_j\right)} \quad \forall i \in E \tag{11}$$

Eq. (11) is equivalent to solve (8) and (9), and provides, together with (10), the solution of the LSFE.

It is necessary to highlight that in (11) it is assumed that all the generators are participating at all the equilibrium points[3] obtained varying $\lambda$, implying that their productions are non null, $P_i > 0$, and therefore $\alpha_i > 0$ [20]. Section 3.1 presents a generalization that allows that some generators be out of the market due to high costs.

## 3. Description of the G-CSFE

In this market formulation the generators' strategies are again assumed to be linear functions (see (1)), but, unlike the LSFE, they are only valid around the equilibrium point (first order Taylor approximations [2]).

Since slopes $\alpha_i$ are not null at the equilibrium point ($\partial P_i(\pi_i)/\partial \pi_i|_{\pi_i=\lambda} = \alpha_i > 0$ for the generators that participate at the equilibrium, see previous section), then:

$$\frac{\partial B_i(\lambda)}{\partial \lambda} = \frac{\partial B_i(\lambda)}{\partial P_i} \cdot \frac{\partial P_i(\lambda)}{\partial \lambda} = 0 \iff \frac{\partial B_i(\lambda)}{\partial P_i} = 0 \quad \forall i \in E \tag{12}$$

[3] A generator participates at an equilibrium point when its production is not affected by active constraints or is not null because of high costs.