Electrical Power and Energy Systems 58 (2014) 291–299

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

A load flow method for weakly meshed distribution networks using powers as flow variables

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ARTICLE INFO

Article history:
Received 1 April 2013
Received in revised form 7 January 2014
Accepted 18 January 2014

Keywords:
Load flow
Weakly meshed network
PV node
Node-branch incidence matrix
Thevenin's equivalent circuit

ABSTRACT

The relationship between the branch powers and the node-injection powers were developed with the node-branch incidence matrix. Then based on two accurate formulas to estimate the voltage drop and angle difference, a new load flow algorithm for weakly meshed distribution systems was presented. By using active and reactive power rather than complex currents as flow variables, the algorithm reduces the computational complexity and has higher efficiency. Moreover, in order to deal with the PV nodes, a new solution was introduced based on Thevenin's equivalent circuit. The solution deduced an accurate calculation formula to update the reactive power injections of PV nodes at each iteration and to fix voltage magnitude of PV nodes at specified values. The proposed load flow algorithm is essentially still belongs to the loop-analysis based method and has a strong ability to deal with meshed network. It reduces the iteration number and has a faster calculation speed even when network becomes more meshed and has more PV nodes. The numerical tests proved that the new method is robust and has excellent convergence characteristics.

1. Introduction

Load flow method as a fundamental tool for distribution management system plays a vital role in evaluation of network condition and optimal operation. Several efficient load flow solution techniques have been developed during the past decades [1–4]. The forward/backward sweep based method is one of the most commonly used load flow solutions in distribution system. It takes advantages of the distribution network structure and has fast calculation speed for no large matrix calculation, and the programming is relatively simple.

For meshed network, the most commonly used solution is the compensation based technique [5]. The method has high calculation speed and good convergence, but it has more iteration number and becomes less efficient when node voltage is lower or when network become more meshed. In [6], an improved version of the method in [5] has been presented, which reduced the related computational effort by using the branch power flows instead of the branch complex currents as flow variables. But it constructs the sensitivity matrix to deal with weakly meshed networks and PV nodes by assuming all bus voltages being close to 1.0 p.u., so its convergence is affected and becomes bad. Ref. [7] presents a novel forward/backward sweep algorithm for meshed network which is firmly based on loop analysis theorem. The method has excellent convergence characteristics even when the network becomes more meshed. Ref. [8] proposed a new load flow method based on the superposition principle according to the structure feature of the weakly meshed network. The method, which can be viewed as a voltage compensation method, is much complicated and not universal. Ref. [9,10] put forward their methods for weakly meshed distribution load flow solutions. The concise formula to formulate the relationship between branch currents and node injections is developed and is used to solve load flow problem directly. The two methods all have higher calculation efficiency and speed, but the method in [10] has a clear theory foundation and a more universal form.

The emergences of distributed generation (DG), which are most based on renewable energy sources, into the distribution systems have great influence on the voltage quality, load flow and power loss, etc. [11]. So far, many approaches have been studied to deal with DG. In [5,6,12], the breakpoint compensation based methods are used to solve radial and weakly meshed systems with PV nodes and obtain good results. But the convergence number increases significantly and sometimes the load flow does not converge when network become more meshed and has more PV nodes. In [10,13], in order to fix voltage magnitude of PV nodes at specified values, two approaches have been presented to cope with PV nodes by updating the reactive power injections of PV nodes at each iteration. The two methods all have good convergence. Ref. [14]

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http://dx.doi.org/10.1016/j.ijepes.2014.01.015
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Nomenclature

N	number of the independent nodes in the network
l	number of link branches (loops) in the network
b	number of the total branches in the network, $b = N + l$
$\tilde{\mathbf{A}}$	node-branch incidence matrix in the network
a_{ij}	element for row i and column j in $\tilde{\mathbf{A}}$
$\mathbf{A}$	reduced node-branch incidence matrix in the network
$\mathbf{A}_{\mathrm{t}}$	reduced node-branch incidence matrix corresponding to the tree branches in $\mathbf{A}$
$\mathbf{A}_{l}$	reduced node-branch incidence matrix corresponding to the link branches in $\mathbf{A}$
$\mathbf{T}_{\mathrm{t}}$	path matrix corresponding to tree branches in the network,
$\boldsymbol{T}_{\mathrm{t}i}$	node i corresponding row vectors (path vector) in $\boldsymbol{T}_{\mathrm{t}}$
$\mathbf{B}_{\mathrm{t}}$	loop matrix corresponding to tree branches in the network,
$\mathrm{d}U$	branch voltage drop
$\mathrm{d}\boldsymbol{\alpha}$	branch voltage angle difference
$\mathrm{d}P$	branch active power loss
$\mathrm{d}Q$	branch reactive power loss
$\boldsymbol{P}$	node consumption active power vector
$\boldsymbol{Q}$	node consumption reactive power vector
$\boldsymbol{P}_{\mathrm{l}t}$	tree branch input active power vector
$\boldsymbol{Q}_{\mathrm{l}t}$	tree branch input reactive power vector
$\boldsymbol{P}_{\mathrm{l}l}$	link branch input active power vector
$\boldsymbol{Q}_{\mathrm{l}l}$	link branch input reactive power vector
$\mathrm{d}\boldsymbol{U}$	branch voltage drop vector
$\mathrm{d}\alpha$	branch voltage angle difference vector
U_{n}	node voltage magnitude vector
$\boldsymbol{\alpha}_{\mathrm{n}}$	node voltage angle vector
$\boldsymbol{I}_{l}$	loop complex current (link branch current) vector
$\boldsymbol{U}_{l}$	loop complex voltage vector
$\boldsymbol{Y}_{l}$	loop admittance matrix
$\boldsymbol{Z}_{\mathrm{b}}$	branch impedance diagonal matrix
$\boldsymbol{Z}_{\mathrm{bt}}$	tree branch impedance diagonal matrix
$\mathrm{d}P_{\mathrm{d}i}$	output active power increment of the ith PV node
$\mathrm{d}Q_{\mathrm{d}i}$	output reactive power increment of the ith PV node

introduces an improved Newton–Raphson algorithm with lower initial values requirement. But the method is much complicated to deal with PV nodes and it needs to calculate the Jacobian matrix. The Jacobian matrix may be ill-conditioned because of the high R/X ratio in distribution system [15]. Ref. [16] presents an improved back/forward sweep load flow method for weakly meshed distribution network with the distributed generation based on the modeling of different DGs (wind turbines, photovoltaic system, fuel cell, CHP, etc.). Ref. [17] proposed a load flow algorithm based on the back/forward sweep method which can deal with PV nodes. The load flow calculation is speeded up by using the laterals delaminating technology.

However, a more efficient method is still needed to deal with weakly meshed networks and PV nodes. This paper presents a new load flow method using powers as flow variables for weakly meshed networks with PV nodes. The main works are the following.

(1) Two accurate formulas to estimate the voltage drop and angle difference have been deduced across a branch through knowledge of terminal power (both active and reactive) and voltage states.
(2) Based on the node-branch incidence matrix, a new load flow solution has been proposed by using powers as variables.
(3) In order to deal with PV node, a new solution is presented. It can be easily integrated into the proposed load flow algorithm.

The advantage of the proposed method and solution are verified by some numerical tests.

2. Formulation of voltage drop and angle difference

The modeling of the serial components in distribution network is used as a basis of most load flow calculation algorithms. A typical distribution line can be modeled as a pure π element which shown in Fig. 1 with related electrical parameters.

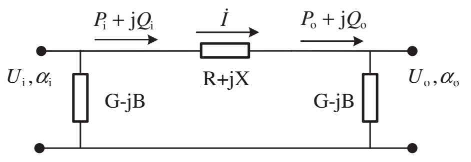

Fig. 1. Equivalent π circuit of distribution lines.

Define voltage drop and angle difference as

$$\begin{cases} \mathrm{d}U = U_{\mathrm{i}} - U_{\mathrm{o}} \\ \mathrm{d}\alpha = \alpha_{\mathrm{i}} - \alpha_{\mathrm{o}} \end{cases} \tag{1}$$

And there exists (superscript $*$ indicates complex conjugate)

$$\dot{I} = \left(\frac{P_{\mathrm{o}} + \mathrm{j}Q_{\mathrm{o}}}{\dot{U}_{\mathrm{o}}}\right)^{*} = \left(\frac{P_{\mathrm{i}} + \mathrm{j}Q_{\mathrm{i}}}{\dot{U}_{\mathrm{i}}}\right)^{*} \tag{2}$$

For simplicity, the shunt impedance $\mathrm{G} - \mathrm{jB}$ is regarded as the impedance-constant load. Their expended power (both active and reactive) can be obtained with terminal voltage as $\mathrm{P_{Gi}} = GU_{\mathrm{i}}^2$, $\mathrm{Q_{Gi}} = -BU_{\mathrm{i}}^2$ and $\mathrm{P_{Go}} = GU_{\mathrm{o}}^2, \mathrm{Q_{Go}} = -BU_{\mathrm{o}}^2$. During calculating the load flow, those powers expended by $\mathrm{G} - \mathrm{jB}$ can be merged into the loads at the corresponding node. It is possible to deduce the accurate formula to calculate difference in voltage magnitude across a branch through knowledge of terminal power (both active and reactive) and voltage states.

In Fig. 2, a vector diagram is used to illustrate interdependencies between input and output voltage magnitude and angle. Apparently, here the output voltage, $\dot{U}_{\mathrm{o}} = U_{\mathrm{o}}\angle 0^{\circ}$, has been taken as the reference vector. The following equation can be obtained when the vector correlations are examined based on Fig. 2,

$$\begin{aligned} \dot{U}_{\mathrm{i}} &= U_{\mathrm{o}} + (\mathrm{R} + \mathrm{jX})\dot{I} = U_{\mathrm{o}} + (\mathrm{R} + \mathrm{jX})\frac{(P_{\mathrm{o}} - \mathrm{j}Q_{\mathrm{o}})}{U_{\mathrm{o}}} \\ &= U_{\mathrm{o}} + \frac{(\mathrm{R}P_{\mathrm{o}} + \mathrm{X}Q_{\mathrm{o}})}{U_{\mathrm{o}}} + \mathrm{j}\frac{(\mathrm{X}P_{\mathrm{o}} - \mathrm{R}Q_{\mathrm{o}})}{U_{\mathrm{o}}} \end{aligned} \tag{3}$$

Moreover, a simple change of taking the input voltage, $\dot{U}_{\mathrm{i}} = U_{\mathrm{i}}\angle 0^{\circ}$, as the reference vector will lead to a completely new diagram and a new equation, as shown by Fig. 3 and (4).

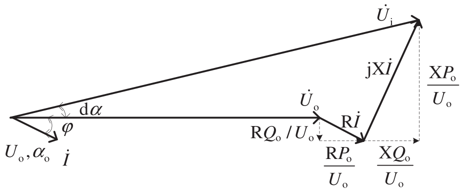

Fig. 2. Input and output voltages vector diagram for equivalent π-model using $\dot{U}_{\mathrm{o}}$ as the reference.