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A robust optimization approach to wind farm diversification

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ABSTRACT

Wind farm diversification can smooth out the fluctuations in wind power generation and reduce the associated system balancing and reliability costs. Recent research has shown that wind farm diversification can be approached using mean–variance method. Traditional mean–variance wind farm diversification method is sensitive to the input data. To overcome the problem of lack of robustness, this paper proposes a novel wind farm diversification method based on robust optimization model. Under box and ellipsoidal uncertainty structures, the proposed robust optimization model can be formulated as a coupled problem composed of a linear programming problem and a conic quadratic programming problem. This model could be efficiently solved. Case studies are provided to demonstrate application of the model.

1. Introduction

With the rapid development of world economy, people have paid more attention to the consumption of fuel and the protection of environment than ever before. Wind power is one of the world's largest and most accessible sources of renewable energy. Compared to the environmental impact of traditional energy sources, the environmental impact of wind power is relatively minor in terms of pollution. Wind power consumes no fuel, and emits no air pollution, unlike fossil fuel power sources. Many countries have adopted the policies prioritizing high wind power penetration [1–3].

However the intermittent and uncertain nature of wind power generation currently prevents it from being widely adopted within national electricity systems. How to overcome the problem of wind power variability is the main challenge for wind energy development. An efficient way to solve this problem is to smoothing out the overall wind power output, in order to reduce its variability and make it more predictable. To some extent, geographical diversification of wind energy capacity can help to flatten this variability [4–8]. The larger geographical area is considered, the more significant this effect is shown, leading further to the possibility of increasing the amount of wind power installed into the system [9–10].

Kahn [4] was the first to systematically analyze these effects for arrays of wind farms of different sizes. In estimating the increased reliability of spatially separated wind plants, Kahn pointed out that "wind generators can displace conventional capacity with the reliability that has been traditional in power systems". Milligan [5] proposed an algorithm based on production costing/reliability methods to find the most reliable allocation of wind power over various wind farms. In particular, Archer and Jacobson [6] pointed out that wind farm diversification would produce "steady deliverable power". They found that "an average of 33% and a maximum of 47% of yearly averaged wind power from interconnected farms can be used as reliable, baseload electric power". Furthermore, mean–variance portfolio theory was extensively applied to wind farm diversification by Drake and Hubacek [11]. They analyzed the average power and standard deviation of several allocations of capacity among four simulated wind farms in the UK to find the allocation with the least amount of wind power variability. Roques et al. [12] and Yannick et al. [13] applied mean–variance portfolio theory to identify cross-country wind farm diversification that reduced the variability of wind power. But none of them detailed their approaches about how they used the data [11–13]. Additionally, Degeilh and Singh [14] introduced a mean–variance framework based on Lagrange multipliers theory to optimize the geographical distribution of wind farms. However, in this wind farm diversification framework, the optimal wind power distribution is very sensitive to the mean and covariance matrix.

This paper focuses on the problem proposed by Degeilh and Singh [14]. To overcome the problem of lack of robustness, we propose a novel wind farm diversification method via robust optimization model. The main contributions of this paper are threefold. First, we define the concept of conditional risk available energy. Accordingly, we study wind farm diversification through

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maximizing the conditional risk available energy. Second, we apply robust optimization techniques to handle the input wind power data uncertainty. Third, we illustrate the efficaciousness and robustness of our proposed model using the IEEE-RTS test system.

The presentation in the paper proceeds as follows. In the second section, the new ideas promoting a diversification of the wind farms are discussed and formalized mathematically. In the third section, a robust optimization under wind power uncertainty process is presented and implemented. Moreover, the tractable formulations under box and ellipsoidal uncertainty sets are described. In the fourth section, the case studies are presented to validate the proposed approach. The last section is the conclusion.

2. A novel wind farm diversification method

The diversification of wind farms studied in the recent papers shares many similarities with that of an investment portfolio, since diversifying approaches for both are developed from the mean–variance theory [11–14]. The basic idea is to avoid a risky dependence on only one source of power/profit because of its unpredictability. In mean–variance analysis framework, risk is defined as the variance of wind power. The optimal wind power distribution is obtained by minimizing the variance of the sum of the various wind farm power outputs. However, when maximizing the part of the wind energy that may provide a stable baseload, the value at risk of the overall energy production is more important than its variance. Therefore, Grothe and Schnieders [15] focused on the allocation of wind turbines by maximizing value at risk of the power supply instead of minimizing the variance. Following this point of view, our study focuses on the allocation of wind energy capacity through maximizing the conditional risk available energy, instead of minimizing the variance of the energy. At the same time, uncertainty of input parameter based on robust optimization will be considered in the next section.

The definition of conditional risk available energy follows from the idea of conditional value at risk [16–17] and conditional robust profit [18–19]. Let $f(x, w)$ denote the wind energy production function with decision vector $x \in X \subseteq R^n$ and wind power output vector $w \in R^m$. If the density function of w is given by $p(w)$, then the probability of $f(x, w)$ not falling below a threshold α is represented as

$$\psi(x,\alpha) = \int_{f(x,w)\geqslant\alpha} p(w)dw \tag{1}$$

and the risk available energy(RAE_β) for a given confidence level β (usually greater than 0.9) is

$$RAE_\beta(x) = \max\{\alpha \in R : \psi(x,\alpha) \geq \beta\} \tag{2}$$

where R is a real set. Consider that the energy is below the $RAE_\beta(x)$, the corresponding conditional risk available energy ($CRAE_\beta$) is

$$CRAE_\beta(x) = \frac{1}{1-\beta}\int_{f(x,w)\leqslant RAE_\beta(x)} f(x,w)p(w)dw \tag{3}$$

From a practical viewpoint, this consideration still makes sense for continuous distributions, since we usually use a discretization procedure to approximate the integral resulting from a continuous distribution. Let the sample space of a random vector w be given by $\{w_{[1]}, w_{[2]}, \ldots, w_{[S]}\}$ with probabilities $P_r\{w_{[k]}\} = \pi_k$ and $\sum_{k=1}^{S}\pi_k = 1$. Denote $\pi = (\pi_1, \pi_2, \ldots, \pi_S)^T$. The auxiliary function is defined as

$$G_\beta(x,\alpha,\pi) = \alpha + \frac{1}{1-\beta}\sum_{k=1}^{S}\pi_k[f(x,w_{[k]})-\alpha]^- \tag{4}$$

where $[f(x, w_{[k]}) - \alpha]^- = \min\{f(x, w_{[k]}) - \alpha, 0\}$. Then we have the formula

$$CRAE_\beta(x) = \max_{\alpha\in R} G_\beta(x,\alpha,\pi) \tag{5}$$

3. Robust optimization for wind power uncertainty

The wind farm diversification model in (5) assumes exact knowledge of the density function $p(w)$. This assumption may not be realistic, particularly in cases where enough data samples are not available, or when the data samples are unstable. Instead of assuming that the random vector w obeys a specific distribution, we assume that the distribution of w is to be determined from the candidates of a set of distributions. Let $\wp_\pi$ denote $\wp$ in the sense of discrete distribution. Instead of assuming a given π, we assume π belongs to a uncertainty set $\wp_\pi$. The robust optimization approach targets the handling of the data uncertainty. It has been proved to be an effective method in practice [20,21].

3.1. Robust optimization

In mathematical optimization, we generally assume that the data is precisely known. We then seek to maximize (or minimize) an objective function over a set of decision variables as follows:

$$\max\ f_0(x, D_0) \tag{6}$$

subject to

$$f_i(x, D_i) \leqslant 0 \quad \forall\ i \in I$$

where x is the vector of decision variables and D_i, $i \in I \cup \{0\}$ are the data that is part of the inputs of the optimization problem.

The input parameters in many real-world optimization problems are uncertain. The robust optimization approach is presented to handle the input data uncertainty. The framework of robust optimization is [22,23]:

$$\max \min_{D_0\in U_0} f_0(x, D_0) \tag{7}$$

subject to

$$f_i(x, D_i) \leqslant 0 \quad \forall\ i \in I,\ \forall\ D_i \in U_i$$

where U_i, $i \in I \cup \{0\}$, are the given uncertainty sets.

Under robust optimization framework, the robust conditional risk available energy ($RCRAE_\beta$) for fixed $x \in X$ with respect to $\wp_\pi$ is defined as

$$RCRAE_\beta(x) = \max_{\alpha\in R}\min_{\pi\in\wp_\pi} G_\beta(x,\alpha,\pi) \tag{8}$$

Particularly, if $\wp_\pi$ is a compact convex set, the problem of maximizing $RCRAE_\beta(x)$ can be written as

$$\max \quad \theta \tag{9}$$

subject to

$$\theta \leqslant \min_{\pi\in\wp_\pi} \alpha + \frac{1}{1-\beta}\sum_{k=1}^{S}\pi_k u_k \tag{9.1}$$

$$u_k \leqslant f(x, w_{[k]}) - \alpha \tag{9.2}$$

$$u_k \leqslant 0, k = 1, 2, \cdots, S \tag{9.3}$$

$$x \in X \tag{9.4}$$

The above problem could not be directly solved by classical optimization theory because of the min operation involved in the constraints. In the following, tractable formulations can be obtained in cases where $\wp_\pi$ is assumed to be a box or ellipsoidal uncertainty set.