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A multiphase optimal power flow algorithm for unbalanced distribution systems

Leandro Ramos de Araujo *, Débora Rosana Ribeiro Penido, Felipe de Alcântara Vieira

Department of Electrical Engineering, Federal University of Juiz de Fora, Juiz de Fora, MG, Brazil

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ABSTRACT

This paper describes a new methodology for the optimization of an *n*-conductor electrical system, in which the phase imbalances, different types of loads, neutral cables, groundings and other inherent characteristics of distribution systems are taken into account. In addition, the methodology is useful for the detailed analysis required for smart grids. A formulation for the optimal power flow of an *n*-conductor system was developed using a primal–dual interior point method and the *n*-conductor current injection method in rectangular coordinates. Distribution and transmission systems were analyzed to verify the generality and efficiency of the proposed methodology.

1. Introduction

With the increase in the distributed generation of distribution systems and the new paradigm of smart grids, new computational tools and better equipment models that enable the representation of electrical networks in detail are needed; indeed, with more-accurate results, more-detailed analyses can be performed [1]. The use of tools that represent such systems as close to reality as possible, incorporating the specified demands of each system, will allow for a much more accurate depiction of the systems and thus will allow the systems to be properly optimized, taking full advantage of their potential. Currently, for smart grids, this need for optimization is greater than ever.

Thus, it is highly important to use tools for the optimization of multiphase electrical systems to determine the conditions of optimal operation from a given objective function, always obeying the restrictions and operational capacity of the equipment and accounting for imbalances. Some optimal power flow algorithms for distribution systems have been based on methodologies previously developed for EHV and UHV transmission networks [2–12], and a few algorithms have been developed for three-phase systems [13–15] but none for multiphase systems.

In [16,17], two power flow methodologies based on the current injection method are presented, which use the Newton–Raphson method to solve for unbalanced three-phase systems. It was observed that the solutions obtained using single-phase or sequence tools to solve unbalanced three-phase problems does not depict the actual operating conditions of electrical systems and may not be sufficient, depending on the level of detail required and the purpose of the study to be conducted.

In [18], a methodology for solving *n*-conductor electrical systems was presented that provided important advances over previous methodologies. All of the advances are detailed in the work mentioned; however, we highlight the following: the methodology does not use the submatrices from previous current injection methodologies; it enables the representation of systems with *n*-conductors; it can represent any type of equipment and their connections and therefore utilizes dimensions strictly necessary for representing a given system. Its greatest advantage is that it is a general method.

Considering the facts presented, there is a need to also develop a multiphase formulation for the optimization of electrical systems, especially one that permits a detailed representation of the characteristics of distribution systems containing *n*-conductors, in addition to enabling joint studies between transmission, subtransmission and distribution and more complete analyses for smart grids. Thus, considering all of the advances obtained in [18], a methodology for optimization was developed, which was named Multiphase Optimal Power Flow (MOPF). The method applies the technique of primal–dual interior points to the model of *n*-conductor power flow. The MOPF will be presented in the following sections.

The organization of the paper is described next. The MOPF is developed in Section 2. Component models in MOPF are presented in Section 3. Objective functions are proposed in Section 4 and a

* Corresponding author. Tel./fax: +55 32 2102 3442.
E-mail address: leandroraraujo@gmail.com (L.R. Araujo).

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MOPF algorithm in Section 5. Section 6 provides three numerical examples a and conclusions are presented in Section 7.

2. Multiphase optimal power flow

The basis of the proposed methodology is the current injection method. By analyzing the current injection equations, it can be concluded that in each node of the system, the current injection sum is composed of parts related to all elements connected to that node; it can also be concluded that each system element provides a current injection contribution to the nodes to which it is connected [18]. To illustrate, the sum of the currents, considering both real and imaginary parts, at node k, a of Fig. 1 is indicated by:

$$\begin{aligned} I_{\mathrm{Re},ka} &= I_{\mathrm{Re},trf,ka} + I_{\mathrm{Re},lin,ka} = 0 \\ I_{\mathrm{Im},ka} &= I_{\mathrm{Im},trf,ka} + I_{\mathrm{Im},lin,ka} = 0 \end{aligned} \tag{1}$$

The two main MOPF constraint equations are for the real and imaginary parts of the currents (I_{Re} and I_{Im}) and are written in rectangular coordinates, where the variables are the real and imaginary parts of the nodal voltages (V_{Re} and V_{Im}); the active and reactive power injections (P and Q) and the appropriate variables arising from control devices ($\mathbf{w}$). In general, the MOPF problem can be defined by:

$$\begin{aligned} &\min\ f(\mathbf{z}) \\ &\text{s.a} \\ &\mathbf{I}_{\mathrm{Re}}(\mathbf{z}) = 0 \\ &\mathbf{I}_{\mathrm{Im}}(\mathbf{z}) = 0 \\ &\mathbf{g}(\mathbf{z}) = 0 \\ &\mathbf{h}(\mathbf{z}) \leqslant 0 \\ &\mathbf{z}_{\min} \leqslant \mathbf{z} \leqslant \mathbf{z}_{\max} \end{aligned} \tag{2}$$

where $f(\mathbf{z})$ is a specific function that should be optimized, $\mathbf{I}_{\mathrm{Re}}(\mathbf{z}) = 0$ and $\mathbf{I}_{\mathrm{Im}}(\mathbf{z}) = 0$ are the sums of the currents injected in the nodes of the system, $\mathbf{g}(\mathbf{z}) = 0$ represents the other equality constraints, $\mathbf{h}(\mathbf{z}) \leqslant 0$ represents the inequality constraints, $\mathbf{z}$ are the decision variables of the problem (state and control) and $\mathbf{z}_{\min} \leqslant \mathbf{z} \leqslant \mathbf{z}_{\max}$ are the lower/upper variable limits.

In this paper, the inequality constraints are transformed into the equality constraints through the use of slack variables. The results obtained are presented in:

$$\begin{aligned} L(\mathbf{z},\boldsymbol{\lambda},\boldsymbol{\pi}) = f(\mathbf{z}) &- \sum_{i=1}^{nn}\lambda_{\mathrm{Im},i} I_{\mathrm{Re},i}(\mathbf{z}) - \sum_{i=1}^{nn}\lambda_{\mathrm{Re},i} I_{\mathrm{Im},i}(\mathbf{z}) - \sum_{i=1}^{ni}\lambda_i g_i(\mathbf{z}) \\ &- \sum_{j=1}^{nd}\pi_{j,1}(z_j - z_{j,\min} - s_{j,1}) - \mu\sum_{j=1}^{nd}\log(s_{j,1}) \\ &- \sum_{j=1}^{nd}\pi_{j,2}(z_j - z_{j,\max} + s_{j,2}) - \mu\sum_{j=1}^{nd}\log(s_{j,2}) \end{aligned} \tag{3}$$

where nn is the number of nodes in the electrical system, ni is the number of equality constraints plus the number of inequality constraints, nd is the number of inequality constraints and lower/upper variable limits, λ_{Re}, λ_{Im}, π_1 and π_2 is the Lagrange multipliers (dual variables), s is the Slack variables associated with the inequality constraints and lower/upper variable limits, μ is the barrier parameter ($\mu > 0$).

It should be noted that in MOPF the contributions of the injected currents in the nodes to the Lagrange function are also written as the individual input contributions of each element. To illustrate, the contributions related to node k, a in Fig. 1 to the Lagrange function are shown in:

$$\begin{aligned} L_{ka}(\mathbf{z}) = -I_{\mathrm{Im},trf,ka}(\mathbf{z})\cdot\lambda_{\mathrm{Re},ka} - I_{\mathrm{Im},lin,ka}(\mathbf{z})\cdot\lambda_{\mathrm{Re},ka} - I_{\mathrm{Re},trf,ka}(\mathbf{z}) \\ \cdot\lambda_{\mathrm{Im},ka} - I_{\mathrm{Re},lin,ka}(\mathbf{z})\cdot\lambda_{\mathrm{Im},ka} \end{aligned} \tag{4}$$

In a multiphase formulation, the contribution of each element is useful because the models are quite different due to the various possibilities of connections (e.g., transformers) and neutral, grounding and mutual impedances. Often, information regarding other phases is needed to define the injection of current in a given phase [18]. Thus, the total contributions of each element were considered throughout the MOPF formulation.

To assemble the Hessian matrix (Jacobian matrix of the optimality conditions) and solve the linear system, the procedure described in [5] was used, in which only the primal variables and the dual λ variables are explicitly represented in the Hessian matrix. The π and s variables are updated each iteration externally to the Hessian matrix. The linear system that should be solved in each iteration is given by:

$$\nabla^2 L(\mathbf{z},\boldsymbol{\lambda})\cdot\Delta[\mathbf{z},\boldsymbol{\lambda}] = -\nabla L(\mathbf{z},\boldsymbol{\lambda}) \tag{5}$$

The structural form of the Hessian matrix ($\nabla^2 L(\mathbf{z},\lambda)$) used in the proposed methodology is shown in Fig. 2 for a node k.

The independent vector $\nabla L(\mathbf{z},\lambda)$ is composed of the first-order derivatives of the Lagrange function with respect to the primal and dual variables. The sequence of the vector independent variables is the same as that in the Hessian matrix.

In MOPF, for each equipment, it is then required to define its contributions to the Lagrange function, independent vector and Hessian matrix, and all models are based on equations injection currents in rectangular coordinates in the nodes to which the components are connected.

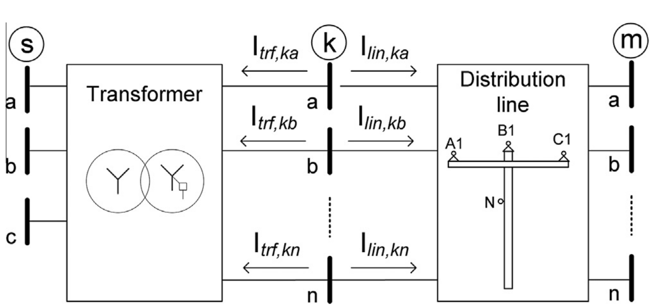

Fig. 1. Example of a multiphase current injections.

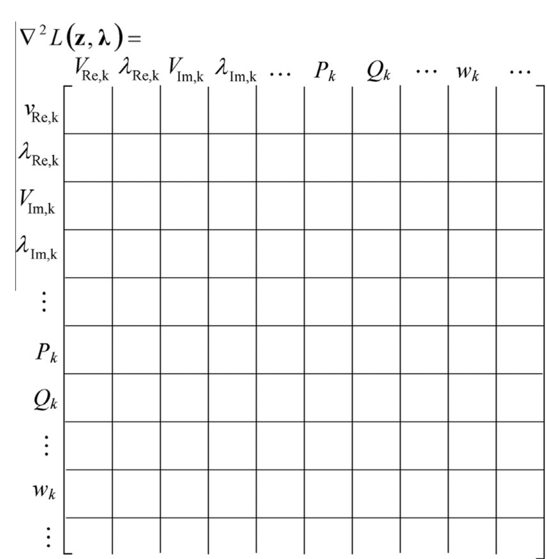

Fig. 2. Structural form of the Hessian matrix.