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Balanced truncation based reduced order modeling of wind farm

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ABSTRACT

This paper applied balanced truncation based model order reduction of wind farms. Starting from a simplified and linearized model of a variable speed wind turbine generator, operating under maximum power tracking, the state space representation of a wind farm is considered. With wind velocities as input, and farm power as the output, the Hankel singular values are computed. These indicate the likely order to which the farm may be reduced to. It is found that reduced model matches well with the initial cluster centers of the distribution of inertia of the WTGs present in the original wind farm. The reduced order model retains the dynamic relationship between the variation in wind speed and consequent power output variations accurately over a wide range of frequencies.

1. Introduction

The increasing penetration of non-conventional and renewable energy based power generation technologies into the grids has increased system complexity. Traditionally, power grid operating scenarios were primarily determined by the temporal evolution of loads. However, with the generation mix incorporating renewable energy sources, it is the fluctuations in these resources, that have also started to affect system operating conditions. Among various renewable energy based power generation systems [1], wind turbine generators (WTGs) technologies are being adopted as part of worldwide smart grid initiatives.

Although wind is a freely available resource, its availability is not always assured. Wind power is always associated with a degree of uncertainty, and variations in wind velocities frequently occur. These have the potential of disturbing the load-generation balance, essential for maintaining power system reliability and security, unless compensated for by other controllable generation resources. A key to understanding the effect of wind velocity fluctuations on the connected network, is to appropriately model individual WTGs and wind farms containing large number of WTGs. With focus on understanding the grid dynamic behavior occurring over minutes, suitable reduced order models of WTGs and wind farms are essential for this exercise.

In this context, model order reduction (MOR) is an established field of study in linear control theory. MOR aims to arrive at reduced models of systems that retain selected aspects of the dynamic behavior. Dynamic reduction has been applied to power system model reduction since the early 1970s. The methodology was essentially semi-heuristic, based on an engineering approach to retaining the modal properties of the power system [2,3]. One of the most important techniques, to arrive at power system dynamic equivalents, relied on establishing the coherency between generators [4–7]. Generally two types of generator aggregation are well-known. One is classical aggregation and another is detailed aggregation. In classical aggregation the coherent group is replaced by an equivalent classical generator model, while detailed aggregation uses detailed equivalent generator model with an equivalent exciter, governor and stabilizer. Most methods discussed above rely on prior experience with the system, heuristically obtained coherency indices and are associated with significant computational burden. Such methods are more suited for synchronous machine based generators; however, WTGs frequently employ asynchronous generators that may be further connected to the grid via power electronic interfaces. Additionally, unlike turbo generators, WTGs are installed in large numbers as wind farms, with hundreds of individual machines of less than 5 MW rating aggregating to produce hundreds of megawatts of power. A starting point of understanding the effect of such wind farms on the power network is to represent them as a few equivalent generators. A similar philosophy may be applied to aggregate future hybrid generation technologies such as fuel cells, photovoltaics [8,9] and batteries that do not possess rotating machines. There are not sufficient literatures available for dynamic aggregation of wind farm. The current focus of this paper is on Balanced Truncation (BT) based reduced order modeling of wind farm.

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While the modal properties of the system state matrix can indicate stability properties, they give no information on the strength of the input–output relationships existing in the system. From a wind farm perspective, with wind velocities as input, and the total farm power as output, what is required is an MOR technique that retains critical input–output relationships while arriving at the reduced order models. The singular value decomposition (SVD) based BT approach [10] is adopted in this paper. In this approach, the original system is reduced by truncating states that do not contribute to the input–output relationships. The strength of these relationships is given by the Hankel singular values (HSVs), and computing the HSVs for different wind farms reveals their dynamic properties. The BT technique has numerous advantages such as it has a reliable stopping criteria based on the HSV of the reduced order system. Further, the reduced order model matches the original model over a wide range of frequencies, while retaining the stability and passivity of the original system.

WTGs are of two types – fixed speed and variable speed [11]. The variable speed WTGs afford greater flexibility in harnessing power from the wind, hence the WTGs considered in this paper are variable speed type, employing doubly fed induction generators (DFIG). The control strategy employed in the DFIG is to operate it at the maximum power point for every wind velocity [12]. In practice this will be true for wind speeds, less than rated speed (i.e. speed at which the WTG generates rated power), while at wind speeds higher than rated speed, the blades are pitched away from the wind to ensure that the rotor does not over speed. In this paper, it is assumed that the wind speeds are always lower than the rated speed, and hence, the action of the pitch controller is ignored. Accordingly, starting from a simplified model of such a variable speed WTG operating under maximum power tracking condition, a wind farm comprising of hundreds of WTGs is considered for MOR. The reduced order model comprises of equivalent machines, whose dynamic parameters depend on the original wind farm, and this is the main contribution of the paper.

The rest of the paper is organized as follows. Section 2 presents a simplified dynamic model of a DFIG based WTG operating under maximum power point tracking. This model is further linearized, and a state space model of the wind farm is obtained in Section 3. Section 4 introduces the BT algorithm, which is then applied for MOR in Section 5. Section 6 is the concluding section.

2. Maximum power tracking in WTGs

As mentioned earlier, in this paper a variable speed WTG is considered, which typically employs a DFIG based generator. The DFIG controls the power at its terminals by controlling the speed of the rotor. At high wind speeds, the pitch angle is set at a positive value to limit the rotor speed. However, in this paper, it is assumed that the wind speeds are low such that the action of the pitch controller does not come into the picture. In this section, a simplified model of a WTG is developed.

At any given wind speed, v_{wind} (m/s), the power extracted by the WTG is given by

$$P_{turb} = \frac{1}{2}\rho \cdot A_r \cdot C_p(\lambda, \beta) \cdot v_{wind}^3 \quad (1)$$

where P_{turb} is the mechanical power (watt), ρ is the density of air (kg/m^3), A_r is the area swept by the turbine (m^2) and $C_p(\lambda, \beta)$ is the performance coefficient or power coefficient. The magnitude of this coefficient $C_p(\lambda, \beta)$ is determined from the C_p–λ curves at different pitch angles, β [13]. 'λ' is known as tip speed ratio [v_t/v_{wind}], which is the ratio between the blade tip speed v_t and the wind speed v_{wind} (before it is retarded by the rotor). The maximum extraction of power occurs, when $C_p(\lambda, \beta)$ is maximum. This is possible at an optimum tip speed ratio, λ_{opt}, that depends on the mechanical design of the turbine. The per unit tip-speed ratio is

$$\lambda_{pu} = \frac{\lambda_{actual}}{\lambda_{opt}} = \frac{\omega_{tur\_pu}}{v_{w\_pu}} \quad (2)$$

where v_{w_pu} is the per unit wind speed on a base of 12 m/s. The per unit angular speed of the turbine, ω_{tur_pu}, equals the per unit generator rotor speed, ω_{r_pu}. Similarly, the performance coefficient can also be normalized to its optimum value at minimum pitch ($\beta = 0$), C_{p_pu}. The per unit mechanical power developed from the turbine, P_m, is then

$$P_m = k_p C_{p\_pu} v_{w\_pu}^3 \quad (3)$$

where the scaling factor k_p (=0.73 in this study), indicates the maximum turbine output power at base wind speed, 12 m/s. Fig. 1 shows the variation of output mechanical power of WTG with wind speed and as well as the rotor speed.

Operation with C_{p_pu} at its maximum value 1 implies that the WTG is harvesting maximum power from the wind. In this study, the WTG is always assumed to be operating under these conditions. From Eq. (3), the mechanical torque at this operating condition is then

$$T_m = \frac{k_p v_{w\_pu}^3}{\omega_{tur\_pu}} \quad (4)$$

For the WTG to operate under maximum power point tracking (MPPT) conditions [14–16], the generator controls the speed to keep maintain optimum tip speed ratio ($\lambda_{pu} = 1$). The generator accordingly follows the MPPT characteristics (as shown in Fig. 1) and the electrical torque (T_e) developed by it is related to the generator speed as [12]

$$T_e = k_p \omega_r^2 \quad (5)$$

Thus at steady state the per-unit output electrical torque T_e is equals the per-unit mechanical torque, T_m. In further discussions, the per-unit rotor speed, ω_{r_pu} is presented as ω, and per-unit wind speed v_{w_pu} as v_w.

The gray lines represent the turbine mechanical power output at different wind speed, while the solid line represents the generator maximum power tracking curve. Point A indicates operation at base wind speed of 12 m/s. Fig. 2 presents the variable speed WTG block diagram equivalent to Eqs. (1)–(4). The turbine block has two inputs – rotor speed and wind speed; the pitch angle input (β), is

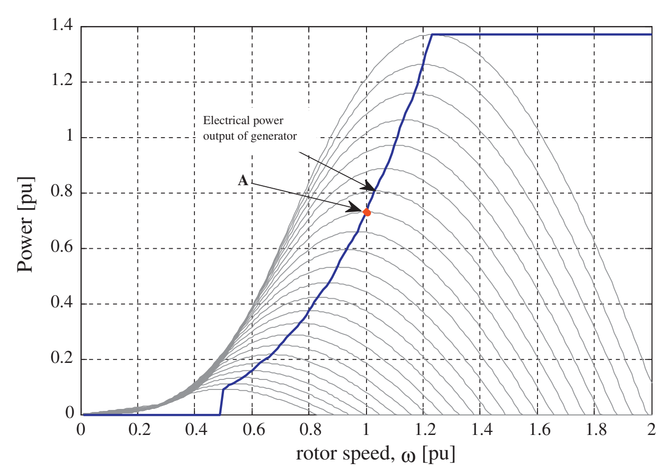

Fig. 1. Power versus rotor speed characteristics for a WTG. The gray lines represent the turbine mechanical power output at different wind speed, while the solid line represents the generator maximum power tracking curve. Point A indicates operation at base wind speed of 12 m/s.