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Time domain transient state estimation using singular value decomposition Poincare map and extrapolation to the limit cycle

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ABSTRACT

This paper proposes an alternative methodology based on the Singular Value Decomposition (SVD) to evaluate the Transient State Estimation (TSE) in the time domain of a power system. Transient phenomena such as faults, sags and load changes can be estimated with the TSE, covering over, normal and under-determined cases, according to the number of measurements and state variables related through the measurement state estimation equation; proposed as being a function of measurements and their derivatives. The system observability can be determined by means of SVD. If the system is unobservable the SVD can determine which parts of the network are observable. The TSE takes partial measurements, calculates the best estimated state variables and completely determines the system state representing a transient condition. The periodic steady state following a transient condition is obtained through a Newton method based on a Numerical Differentiation (ND) process, Poincaré map and extrapolation to the limit cycle; this method is applied before and after the simulated transient. The TSE results are validated through direct comparison against those obtained with the Power Systems Computer Aided Design/Electromagnetic Transients Program including Direct Current (PSCAD/EMTDC) simulator.

1. Introduction

The Power Quality State Estimation (PQSE) is an important topic for the power system operation to deliver the electrical energy with good quality indexes, the TSE can be classified as part of the PQSE [1,2].

The methods to solve the time domain TSE are the recursive normal and weighted least squares, least mean squares [3], SVD, Kalman filter [4], optimization methods [5] and neural networks. The least square methods have disadvantage to solve the state estimation mainly in under-determined cases or ill conditioned matrices; Kalman filter requires the initial previous state and the covariance matrices; the optimization methods and neural networks demand a high computational effort when are applied in the time domain. A method based on the SVD is proposed to solve the TSE in association with the ND method to obtain the periodic steady state of the power system. The SVD advantages are the solution of ill conditioned matrices and under-determined cases also the intrinsic observability analysis during the state estimation [6].

The TSE can be seen as a transient simulation reverse process. Fig. 1 shows this relationship, the main objective is to calculate the best estimates of the state variables under a transient condition where they are not monitored. An electrical system can be state space represented by a first order ordinary differential equations (ODE) set and the output equation. The state estimation measurement equation ($\boldsymbol{z} = \boldsymbol{Hx}$) is formulated based on the state space equations and on the network topology relating the measurements to the state variables; each measurement adds an equation to set up the measurement equation [7,8]. From the TSE results, it is possible to identify the disturbance location by inspecting the current and voltage mismatches within the network. TSE can be used to quickly locate the disturbances to take prompt corrective actions [9,10].

TSE uses a limited number of measurements; these data can be contaminated with noise and can possibly have gross measurement errors. The choice of measurement points and quantities to measure are important aspects to take into account as this will influence the system observability [12] and if the measurement equation is over, normal or under-determined [8,13].

TSE estimates fluctuations in the waveforms; one approach is to use the criterion of minimizing the squared error sum between the measured and estimated values (least square estimation LSE). This error indicates the state estimation accuracy [14,15]. The state space formulation matrices are used to follow the system dynamics in the time domain and to evaluate the state and output variables; with these values any variable in the system can be calculated [16].

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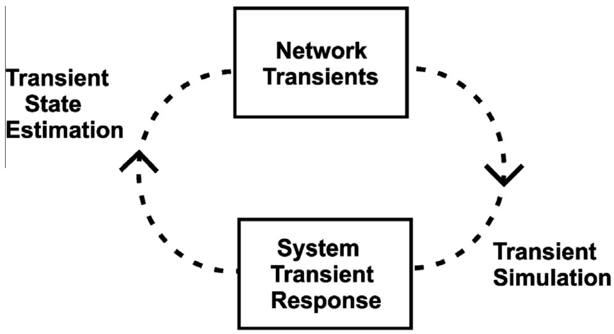

Fig. 1. Reverse process relating the transient simulation and transient state estimation [11].

The conventional state estimation includes in its measurement vector, nodal voltages, power flows, power injections and current magnitudes. The measurement equation is nonlinear and demands an iterative solution. If the measurement equation is linear, as in TSE, a direct solution is performed. In state estimation a useful criterion is the weighted least squares; this can be used with the normal equation to solve the estimation, but this formulation fails when the measurement matrix is ill conditioned; under this condition an alternative is to use the SVD [16].

A Newton method based on a ND process, Poincaré Map and extrapolation to the limit cycle is used to find the periodic steady state before and after the transient state, the TSE is solved under the periodic steady state to be sure that only one transient is present in the network; the ND method reduces the execution time and the computational effort of the simulated transient mainly when the considered system has long time constants or is under-damped, this is possible due to the reduction of the cycles to be processed [17].

The rest of the paper is organized as follows: Section 2 details the proposed methodology for the TSE; with three subsections to explain the SVD decomposition, the ND method and the numerical derivative. Section 3 presents the case studies, for over, normal and under-determined cases to verify the TSE. Section 4 presents the main conclusions drawn from this research work.

2. Methodology

The methodology includes the next four steps to solve the time domain TSE using the SVD and the ND method,

(1) Definition of $\boldsymbol{H}$ measurement matrix selecting output variables and their derivatives as measurements and SVD decomposition of $\boldsymbol{H}$ to define the observability and the system condition.
(2) ND determines the periodic steady state for the network to remove the initial transient and obtains a more convenient initial condition to evaluate the TSE avoiding divergence problems.
(3) SVD solves the TSE including over, normal and under-determined cases. The estimated state variables, measurements and the physical laws are used to evaluate the rest of variables in the network.
(4) ND obtains the new periodic steady state of the power system after the estimated transient condition.

The state space system model is:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \tag{1}$$

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \tag{2}$$

Table 1
Variable options as measurements.

Measurement	System variable	Definition
$\boldsymbol{y}$	$\boldsymbol{Cx} + \boldsymbol{Du}$	Output variable
dy/dt	$\boldsymbol{CAx} + \boldsymbol{CBu} + \boldsymbol{Du}$	Output variable derivative

The state estimation measurement equation can be structured with various options, Table 1 shows the variables that can be taken as measurements [10].

This work proposes to take as measurements a combination of output variables and their derivatives; this ensures a linear measurement equation. If a state variable can be measured according to the proposed model, this variable can be used to set up the measurement equation. The measurements set defines the measurement equation, i.e.,

$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} \tag{3}$$

This equation is solved using the least squares criterion for the estimation error when the $\boldsymbol{H}$ matrix is non-singular, by means of the normal equation formulation or using the SVD [7,9]:

$$\boldsymbol{x} = (\boldsymbol{H}^T\boldsymbol{H})^{-1}\boldsymbol{H}^T\boldsymbol{z} = \boldsymbol{V}\boldsymbol{W}^{-1}\boldsymbol{U}^T\boldsymbol{z} \tag{4}$$

When $\boldsymbol{H}$ is ill conditioned or under-determined, SVD gives a solution using the pseudo-inverse,

$$\boldsymbol{x}^+ = \boldsymbol{V}\boldsymbol{W}^+\boldsymbol{U}^T\boldsymbol{z} \tag{5}$$

Eq. (1) relates the state variables with the system dynamics; it can be linear or nonlinear depending of the system and its components. In this work the linear time invariant case is analyzed. Eq. (3) can be added with a vector v representing the measurements noise [7,9] and is related with the state space output Eq. (2) as,

$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v} \tag{6}$$

The state estimation error is $\boldsymbol{e} = \boldsymbol{z} - \boldsymbol{z}_{estimated} = \boldsymbol{z} - \boldsymbol{H}\boldsymbol{x}_{estimated}$

2.1. Singular value decomposition [18]

The SVD gives a unique solution when the system is over or normal-determined ($m > n$ or $m = n$, m-measurements, n-states), but when the system is under-determined ($m < n$), the SVD can give a solution with minimum norm and establish the system observability. The observable states are estimated and the unobservable areas of a power system can be delimited [19]. SVD can be used to verify the system observability before or during the state estimation [20], [21]. SVD factorizes the measurement matrix $\boldsymbol{H}$, i.e.,

$$\boldsymbol{H} = \boldsymbol{U}\boldsymbol{W}\boldsymbol{V}^T \tag{7}$$

The condition number of a matrix is the division of the largest to the smallest singular value. A matrix with a large condition number is ill conditioned. If a singular value is zero or near zero, a zero is placed in the corresponding diagonal element of W^{-1} instead of $1/w$, then if,

$$\boldsymbol{W} = \begin{bmatrix} \boldsymbol{S} & 0 \\ 0 & 0 \end{bmatrix} \tag{8}$$

Its pseudo-inverse is defined by:

$$\boldsymbol{W}^+ = \begin{bmatrix} \boldsymbol{S}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \tag{9}$$

The pseudo-inverse of $\boldsymbol{H}$ is,

$$\boldsymbol{H}^+ = \boldsymbol{V}\boldsymbol{W}^+\boldsymbol{U}^T \tag{10}$$

This gives the pseudo-inverse solution,

$$\boldsymbol{x}^+ = \boldsymbol{H}^+\boldsymbol{z} = \boldsymbol{V}\boldsymbol{W}^+\boldsymbol{U}^T\boldsymbol{z} \tag{11}$$