

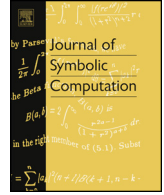


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Flat extension and ideal projection [☆]

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ABSTRACT

A generalization of the flat extension theorems of Curto and Fialkow and Laurent and Mourrain is obtained by seeing the problem as one of ideal projection.

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1. Introduction

The Hamburger moment problem is to determine the existence and uniqueness of a positive measure whose moments

$$\int_{\mathbb{R}^k} x^\alpha d\mu, \quad \alpha \in \mathbb{Z}_+^k,$$

take a prescribed multi-sequence of values $(y_\alpha)_{\alpha \in \mathbb{Z}_+^k}$. Here, x^α is the α th monomial in Π , the vector space of all polynomials on \mathbb{R}^k . For such a measure to exist, it is necessary and sufficient that the linear functional

$$L : \Pi \rightarrow \mathbb{R} : p = \sum_{\alpha} \hat{p}(\alpha) x^\alpha \mapsto \sum_{\alpha} \hat{p}(\alpha) y_{\alpha}$$

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be nonnegative, that is, $Lp \geq 0$ if $p \geq 0$, and for this to occur, it is necessary that the **moment matrix**

$$[y_{\alpha+\beta}]_{\alpha, \beta \in \mathbb{Z}_+^k} \quad (1)$$

be positive semidefinite, that is, $L(p^2) \geq 0$ for all p . This condition is sufficient only under special circumstances, for instance, when every positive polynomial can be written as a sum of squares, as occurs when $k = 1$ (Pólya and Szegő, 1976, §6.6) and again when (1) has finite rank, since in that case the functional L is finitely atomic, i.e., a linear combination of finitely many shifts of δ and its derivatives (Laurent and Rostalski, 2012, Theorem 7).

Finitely atomic measures and their moment matrices are the subject of the **truncated moment problem** of Curto and Fialkow (1996), who address when such a measure is determined by finitely many of its moments. To be precise, Curto and Fialkow work in \mathbb{C}^k and study the relationship between the complex moment matrix

$$\left[\gamma_{\alpha, \beta} := \int_{\mathbb{C}^k} \bar{x}^\alpha x^\beta d\mu \right]_{\alpha, \beta \in \mathbb{Z}_+^k}, \quad (2)$$

its submatrices of the form

$$[\gamma_{\alpha, \beta}]_{|\alpha|, |\beta| \leq n}, \quad (3)$$

and the size of the support of μ . One of their main results is that if the positive semidefinite (3) can be extended **flatly**, meaning without increasing its rank, to a positive semidefinite

$$[\gamma_{\alpha, \beta}]_{|\alpha|, |\beta| \leq n+1},$$

then the later has a unique, positive semidefinite, flat extension (2) and a finitely-atomic representing measure μ , and the rank of the matrix equals the cardinality of the measure's support.

Curto and Fialkow's flat extension theorem in the special case when \mathbb{C} is replaced by \mathbb{R} is generalized by Laurent and Mourrain (2009). Working with moment matrices of the form (1), they replace \mathbb{C} by any field \mathbb{F} and do not require moment matrices to be positive semidefinite. They then prove that if the finite set $C \subset \mathbb{Z}_+^k$ is **connected to one**, meaning

$$C \neq \emptyset, \text{ and } \alpha \in C \setminus 0 \implies \alpha - e_j \in C \text{ for some } j, \quad (4)$$

and if

$$[y_{\alpha+\beta}]_{\alpha, \beta \in C} \quad (5)$$

can be flatly extended to

$$[y_{\alpha+\beta}]_{\alpha, \beta \in C^+} \quad (6)$$

where

$$C^+ := C \cup \{\alpha + e_j : \alpha \in C, j = 1 : k\}$$

and $e_j \in \mathbb{F}^k$ is the vector given by

$$(e_j)_i = \delta_{i,j}, \quad i, j = 1 : k, \quad (7)$$

then (6) has a unique flat extension (1). In case \mathbb{F} is \mathbb{R} , if (6) is positive semidefinite, then so is (1).

(To be consistent with Laurent and Mourrain, we'll say that the monomial space $\Pi_C := \text{span}\{x^\alpha : \alpha \in C\}$ is connected to one if the set $C \subset \mathbb{Z}_+^k$ is connected to one.)

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