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Foreword

Foreword to special issue

1. Overview

This special issue focuses on utilizing the structure of polynomial equations originating from geometric constraint systems, creating a single venue for results that have traditionally spanned several research areas. Geometric constraint solving is concerned with the realization and algebraic dependence problems of a variety of geometric constraint systems (Brüderlin and Roller, 1998). The realization problem seeks to characterize and navigate the (typically real) solution space of a geometric constraint system, where the constraints are expressed as polynomial equations and chosen from an extensive repertoire (Sitharam et al., 2006). The dependence problem aims to detect constraints that are dependent or implied by (sets of) others.

Several research areas approach these problems from different perspectives. Combinatorial rigidity focuses on properties of the underlying (hyper)graphs of geometric constraint systems that characterize when a generic solution or realization is locally (respectively, globally) rigid, i.e., isolated (respectively, unique) (Graver et al., 1993). Geometric rigidity extends to nongeneric settings and gives geometric characterizations of local and global rigidity as well as other properties of the realization or configuration space of geometric constraint systems (Cauchy, 1813; Dehn, 1916; Aleksandrov, 1958; Hilbert and Cohn-Vossen, 1952). Distance geometry is a classical area that focuses on metric constraint systems and uses the convexity induced by metric inequalities to infer properties of their solution spaces (Schoenberg, 1938; Deza and Laurent, 2009). In the case of Euclidean distances, the entire area of semidefinite programming becomes a subtopic (Dattorro, 2005).

These research areas rely on a variety of techniques: graphs and matroids, algebraic geometry and topology, synthetic geometry, metric spaces and convexity. Symbolic analysis for structural properties of geometric constraint systems is of interest to a variety of applied domains, including: Computer-Aided Design, Structural Molecular Biology, Sensor Network Location, Robotics, Machine Learning, Geometry-based Teaching.

For a concrete setting of the types of problems we aim to study, consider a classical bar-and-joint framework, composed of a collection of fixed-length bars attached at flexible joints. We model this constraint system as a system of distance equations among a finite set of points in Euclidean space of some fixed dimension.

- The combinatorial rigidity community traditionally focuses on combinatorial approaches for detecting whether an input framework is locally rigid, i.e., whether there is a zero-dimensional real solution set of locally isolated realizations, or globally rigid, i.e., with exactly one realization; in both cases, the realizations are defined modulo "trivial" transformations for Euclidean geometry. A classical approach is to analyze the first-order behavior of the nonlinear incidence constraints, showing that the resulting infinitesimal rigidity is equivalent to rigidity in the generic case (Asimow and Roth, 1979), then study the resulting "rigidity matrix," whose entries are linear forms in

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the indeterminate coordinates of the framework's joints. Structural analysis of the rigidity matrix (e.g., a Laplace decomposition, White and Whiteley, 1987) leads to a combinatorial property of the underlying graph that guarantees full rank of the linear system, provided certain degeneracies expressed through a polynomial called the "pure condition" are avoided. The question of linear dependencies between the rows of the rigidity matrices leads to the notion of rigidity matroids (Whiteley, 1996). Full-blown symbolic approaches are required to study the nongeneric situations (Montes and Wibmer, 2010; White and Whiteley, 1987; Farre et al., 2016).

- The Grassmann–Cayley algebra provides a connection between the equations determining local rigidity and the original geometry, and methods of computational algebra can be brought to bear on the problem of understanding when a framework with given combinatorics is generic or not. Like any Gröbner calculation, the worst-case running time analysis can be costly, but as in algebraic geometry, it may be possible to identify geometric features that permit better bounds on the running time of computations (Cox et al., 2015; Li, 2016).
- The geometric constraint solving community focuses on problems related to finding and describing the realization spaces for input distance constraint systems and understanding how to add constraints to globally or locally rigidify the corresponding framework (force isolated or unique realizations), often combining combinatorial, symbolic and numerical approaches (Elkadi et al., 2006; Jermann et al., 2006; Sitharam, 2005). For the related automated geometry and geometric reasoning community, understanding rigidity can be phrased as determining whether the missing distance equations of a geometric constraint system or framework are implied by (i.e., in the real ideal generated by) the initial set of distance equations. In this bar-and-joint setting, the underlying geometry is Euclidean, and the constraints are pairwise distances, which are polynomial invariants for the Euclidean group of transformations.
- For the distance geometry community, rigidity in a given dimension is a property of a fiber or section of a dimensional stratum of the semidefinite cone of Euclidean distance matrices (Dattorro, 2011), or alternatively of the semi-algebraic set described by Cayley–Menger determinantal inequalities.

There are many variations of geometric constraint systems and frameworks beyond the classical bar-and-joint frameworks (including body-and-bar, slider-and-pin, and body-and-cad frameworks, arising in the above-mentioned application domains) which are amenable to similar analyses. Some of these systems are best interpreted in non-Euclidean underlying geometries, including similarity, affine and projective geometries.

We hope this special issue will help researchers working in the areas listed above to see themselves as part of a cohesive community. We envision that increased interaction between these groups will help the development of

- approaches for analyzing systems of increasingly varied and rich systems of geometric constraints that arise in applications
- approaches to problems that are intractable for general polynomial systems that become tractable for geometric systems; i.e., understanding how the geometric underpinnings of a system of constraints affect the feasibility of working computationally with the resulting equations.

2. Contributed papers

The first paper by Panina and Siersma concerns the much-studied configuration space of plane polygonal linkages (Farber, 2008; Streinu, 2000; Connelly et al., 2003), i.e., the solution space of geometric constraint systems given by a cyclic system of distance constraints between point-pairs. It uses the topology of the moduli space (space of solutions modulo the rigid body transformations or Euclidean isometries), to describe an algorithm for obtaining a continuous path from a start to an end configuration. A constant bound is given on the number of relevant iterations needed in the algorithm. The 1-skeleton of moduli space is the focus of the paper and indeed the algorithm presented involves carefully walking along the skeleton to get between successive designated configurations. An analysis of the meaning of the vertices and edges of the moduli space is presented, along with how to