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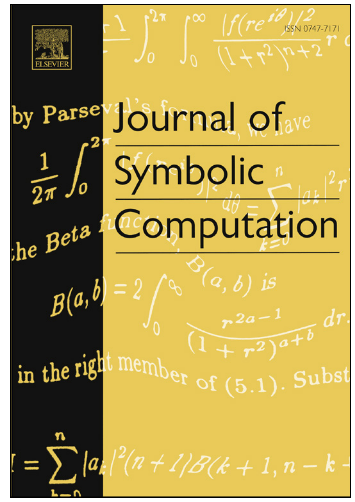

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Triangular bases of integral closures

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Abstract

In this work, we consider the problem of computing triangular bases of integral closures of one-dimensional local rings.

Let (K, v) be a discrete valued field with valuation ring $\mathcal{O}$ and let $\mathfrak{m}$ be the maximal ideal. We take $f \in \mathcal{O}[x]$, a monic irreducible polynomial of degree n and consider the extension $L = K[x]/(f(x))$ as well as $\mathcal{O}_L$ the integral closure of $\mathcal{O}$ in L, which we suppose to be finitely generated as an $\mathcal{O}$-module.

The algorithm MaxMin, presented in this paper, computes triangular bases of fractional ideals of $\mathcal{O}_L$. The theoretical complexity is equivalent to current state of the art methods and in practice is almost always faster. It is also considerably faster than the routines found in standard computer algebra systems, excepting some cases involving very small field extensions.

1. Introduction

Let (K, v) be a discrete valued field with valuation ring $\mathcal{O}$. Let $\mathfrak{m}$ be the maximal ideal, $\pi \in \mathfrak{m}$ a generator of $\mathfrak{m}$ and $\mathbb{F} = \mathcal{O}/\mathfrak{m}$ the residue class field.

Let K_v be the completion of K, and retain $v : \overline{K}_v^* \to \mathbb{Q}$ the canonical extension of v to a fixed algebraic closure of K_v. Let $\mathcal{O}_v$ be the valuation ring of K_v.

Let $f \in \mathcal{O}[x]$ be a monic, irreducible polynomial of degree n and fix a root $\theta \in \overline{K}$ in an algebraic closure of K. Let $L = K(\theta)$ be the corresponding finite extension of K and let $\mathcal{O}_L$ be the integral closure of $\mathcal{O}$ in L, which is a Dedekind domain. We denote the set of non-zero prime ideals of $\mathcal{O}_L$ by $\mathcal{P}$.

We suppose that $\mathcal{O}_L$ is finitely generated as an $\mathcal{O}$-module. This condition holds under very natural assumptions; for instance, if L/K is separable, or (K, v) is complete, or $\mathcal{O}$ is a finitely generated algebra over a field (Serre, 1968, Ch.I, §4). Under this hypothesis, $\mathcal{O}_L$ is a free $\mathcal{O}$-module of rank n. An $\mathcal{O}$-basis of $\mathcal{O}_L$ is called a *v-integral basis* of $\mathcal{O}_L$.

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