Journal of Symbolic Computation 67 (2015) 93–121

Contents lists available at ScienceDirect

Journal of Symbolic Computation

www.elsevier.com/locate/jsc

Bottom-up rewriting for words and terms

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ARTICLE INFO

Article history:
Received 4 August 2013
Accepted 15 February 2014
Available online 17 June 2014

MSC:
68Q42
03D03
03D40

Keywords:
Term rewriting systems
Semi-Thue systems
Regularity preservation
Accessibility problem

ABSTRACT

For the whole class of linear term rewriting systems, we define *bottom-up rewriting* which is a restriction of the usual notion of rewriting. We show that bottom-up rewriting effectively inverse-preserves recognizability.
The *Bottom-Up* class (BU) is, by definition, the set of linear systems for which every derivation can be replaced by a bottom-up derivation. Since membership to BU turns out to be undecidable, we are led to define more restricted classes: the classes $\mathsf{SBU}(k)$, $k \in \mathbb{N}$, of *Strongly Bottom-Up*(k) systems for which we show that membership is decidable. We define the class of *Strongly Bottom-Up* systems by $\mathsf{SBU} = \bigcup_{k\in\mathbb{N}} \mathsf{SBU}(k)$. We give a polynomial-time sufficient condition for a system to be in SBU. The class SBU contains (strictly) several classes of systems which were already known to inverse preserve recognizability: the inverse left-basic semi-Thue systems (viewed as unary term rewriting systems), the linear growing term rewriting systems, the inverse Linear-Finite-Path-Ordering systems.

1. Introduction

General framework. An important concept in rewriting is the notion of *preservation of recognizability* through rewriting. Each identification of a more general class of systems preserving recognizability, yields almost directly a new decidable call-by-need (Durand and Middeldorp, 2005) class, decidability results for confluence, accessibility, joinability. Also, recently, this notion has been used to prove termination of systems for which none of the already known termination techniques work (Geser et al., 2005). Such a preservation property is also a tool for studying the

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http://dx.doi.org/10.1016/j.jsc.2014.06.001
0747-7171/

recognizable/rational subsets of various monoids which are defined by a presentation $\langle X, \mathcal{R}\rangle$, where X is a finite alphabet and $\mathcal{R}$ a Thue system (see for example Lohrey and Sénizergues, 2008; Kambites et al., 2007). Consequently, the seek of new decidable classes of systems which preserve (or inverse preserve) recognizability is worthwhile.

Many such classes proposed so far have been defined by imposing syntactical restrictions on the rewrite rules. For instance, in *growing* systems (Jacquemard, 1996; Nagaya and Toyama, 2002) variables at depth strictly greater than 1 in the left-hand side of a rule cannot appear in the corresponding right-hand side. Finite-path Overlapping systems (Takai et al., 2010) are also defined by syntactic restrictions on the rules. The class of Finite-path Overlapping systems contains the class of growing systems (Nagaya and Toyama, 2002). Previous works on semi-Thue systems also prove recognizability preservation, under syntactic restrictions: cancellation systems (Benois and Sakarovitch, 1986), monadic systems (Book et al., 1982), basic systems (Benois, 1987), and left-basic systems (Sakarovitch, 1979) (see Sénizergues, 1995 for a survey).

Other works establish that some *strategies* i.e. restrictions on the derivations rather than on the rules, ensure preservation of recognizability. Various such strategies were studied in Fülöp et al. (1998), Réty and Vuotto (2005), Seynhaeve et al. (1999).

We rather follow here this second approach: we define a new rewriting strategy which we call *bottom-up rewriting* for linear term rewriting systems. The bottom-up derivations are, intuitively, those derivations in which the rules are applied, roughly speaking, from the bottom of the term towards the top (this set of derivations contains strictly the bottom-up derivations of Réty and Vuotto (2005) and the one-pass leaf-started derivations of Fülöp et al. (1998); it is incomparable with the *innermost* derivations[1]). An important feature of this strategy, as opposed to the ones quoted above, is that it allows *overlaps* between successive applications of rules. A class of systems is naturally associated with this strategy: it consists of the systems $\mathcal{R}$ for which the binary relation $\rightarrow^*_{\mathcal{R}}$ coincides with its restriction to the bottom-up strategy. We call “bottom-up” such systems and denote by BU the set of all bottom-up systems.

Overview of the paper. The results proved in this paper were announced in Durand and Sénizergues (2007), which can thus be considered as a medium-scale overview of this paper.

In Section 2, we have gathered all the necessary recalls and notation about words, terms, rewriting and automata.

In Section 3, we define *bottom-up rewriting* for linear term rewriting systems using marking techniques. We first define *bottom-up*(k) derivations for $k \in \mathbb{N}$ ($\mathsf{bu}(k)$ derivations for short) and the classes Bottom-up(k) ($\mathsf{BU}(k)$ for short) of linear systems which consists of those systems which admit $\mathsf{bu}(k)$ rewriting, i.e. such that every derivation between two terms can be replaced by a $\mathsf{bu}(k)$ derivation, and the *Bottom-up* class (BU) of *bottom-up* systems which is the infinite union of the $\mathsf{BU}(k)$ (for k varying in $\mathbb{N}$).

In Section 4, we prove Theorem 4.2 which is the main result of the paper: bottom-up rewriting inverse-preserves recognizability. Our proof consists of a reduction to the preservation of recognizability by finite ground systems, shown in Brainerd (1969), Dauchet and Tison (1990). The proof is constructive i.e. gives an algorithm for computing an automaton recognizing the antecedents of a recognizable set of terms.

It turns out that membership to $\mathsf{BU}(k)$ is undecidable for $k \geq 1$ (Durand and Sénizergues, 2009, Theorem 5.12). In Section 5, we thus define the restricted class of *strongly bottom-up*(k) systems ($\mathsf{SBU}(k)$) for which we show *decidable* membership. We define the class of *strongly bottom-up* systems $\mathsf{SBU} = \bigcup_{k\in\mathbb{N}} \mathsf{SBU}(k)$ and give a polynomial sufficient condition for a system to be in SBU.

2. Preliminaries

This section is mostly devoted to recalling some classical notions and making precise our notation. The reader is referred to Comon et al. (2002) for more details on the subject of tree-automata and to Klop (1992), Terese (2003) for term rewriting.

[1] Which, anyway, do not inverse-preserve nor preserve recognizability.