Knowledge-Based Systems 000 (2018) 1–14

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Fuzzy Bayes risk based on Mahalanobis distance and Gaussian kernel for weight assignment in labeled multiple attribute decision making

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A R T I C L E I N F O

Article history:
Received 15 November 2017
Revised 30 March 2018
Accepted 1 April 2018
Available online xxx

Keywords:
Weight assignment
Fuzzy Bayes risk
Mahalanobis distance
Gaussian kernel
Labeled MADM
Effectiveness evaluation

A B S T R A C T

Attribute weight assignment plays a key role in multiple attribute decision making (MADM). For the issue of labeled multiple attribute decision making (LMADM), the existing methods of attribute weight determination that have been well developed for MADM usually ignore or do not take full advantage of the supervisory function of labels. As a result, the weights produced by these methods may not be ideal in practice. To make up for this deficiency, this paper develops an objective method based on Bayes risk. Specifically, the LMADM problem is first put forward, then a Gaussian kernel based loss function is proposed to cope with the drawback that the loss function in Bayes risk is usually determined by experts. Meanwhile, Mahalanobis distance and fuzzy neighborhood relationship are employed to measure the fuzziness of data set. Finally, a number of experiments, including the comparison experiments on UCI data and the effectiveness evaluation of fighter, are carried out to illustrate the superiority and applicability of the proposed method.

1. Introduction

Data systems in practical applications can be broadly divided into two categories, decision system (DS) and information system (IS). The first category named DS is a set of data consisted of conditional attributes and decision attributes, and the other one called IS does not include decision attributes, i.e., labels. In practical applications, multiple attribute decision making (MADM) is one of the most important and fundamental issues in the field of DSs, which could be called labeled multiple attribute decision making (LMADM), because of its significant applications in various fields such as effectiveness evaluation [1], classification [2] and fault diagnosis [3].

Attribute weight assignment is one of the most significant parts in MADM, which has been studied in-depth in a variety of aspects [4–11]. Generally, it can be classified into three categories of methods, i.e., subjective methods [12,13], objective methods [14–16] and hybrid methods [17–20], according to the extent of dependence on the preferences or subjective judgements of decision makers (DMs) [4]. In practical applications, it is usually quite hard to obtain ideal weight results by using the subjective or hybrid methods when there are lack of related field experts or no unanimous conclusion reached by DMs [21,22]. Fortunately, the objective weight methods can solve the above problem effectively, which generate attribute weights from data alone without requiring any preference information from the DMs. According to the applied data systems, there could be two parts with respect to the objective methods, one is named ISOM (objective method for information system) and the other one is called DSOM (objective method for decision system). However, to the best of our knowledge, most of objective weight assignment methods are aimed at IS, such as Entropy method [4,23,24], Principal Components Analysis (PCA) method [25], criteria importance through inter-criteria correlation (CRITIC) method [16], modifying TOPSIS method [14], standard deviation (SD) and mean deviation (MD) method [26], correlation coefficient and standard deviation (CCSD) method [15] and some other weight assignment methods in different issues (see, e.g., [27–29] and the references therein). Nevertheless, there are few relevant studies on DS. These methods, such as grey relation analysis (GRA) [30] approach, conditional entropy (CE) [31] approach, rough set (RS) [32,33] approach, F-score approach [34] and mutual information approach [35], could be the alternatives for the assignment of conditional attribute weights in DSs, due to the considering of the coupling relationships between conditional attributes and decision attribute.

All of the above methods for ISs, however, do not consider the contributions of decision attributes to the determination of conditional attribute weights, when they are applied to DS. The conditional attributes are the descriptions of the whole system in some concerned aspects. Usually, there is only one decision attribute in

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https://doi.org/10.1016/j.knosys.2018.04.002
0950-7051/

DS,[1] which is a generalization of the overall system and an abstract of all the conditional attributes. Each conditional attribute provides a particular contribution to its system and an individual support degree to the abstract of decision attribute, which could be depicted as the weight of conditional attribute. Therefore, for DSs, the determination of conditional attribute weights cannot ignore the role of decision attribute.

In fact, with regard to MADM, the final decision produced by any decision making unit will be accompanied by some risks. These risks stem from the data distributions of conditional attributes, consequently, each of the conditional attributes will generate a unique risk for the final decision, which could employ the weight of attribute as a metric. However, the exiting methods have not taken the decision risk as a main factor to determine the weights of attributes. On the other hand, the current methods may not take into account the fuzziness of data system, which includes two aspects: one is the fuzziness among the samples and the other one is that between the samples and the decision classes. Therefore, there are two kinds of coupling relations between the samples induced by conditional attributes and decision attribute, i.e., the decision risk and the fuzzy membership. Further more, with respect to the weight assignment of multi-layer attribute set, it usually needs the help of experts/DMs, or it is achieved through some complex combination methods [17], which greatly limits the application of weight determination methods in multi-layer index system. These inadequacies of the present researches motivate this work.

To handle the aforementioned issues and overcome the deficiencies of the existing methods, we propose a simple and effective objective attribute weight assignment method (MGFBRW) using Mahalanobis distance and Gaussian kernel based fuzzy Bayes risk (MGFBR) method, which is applicable not only to ISs and DSs, but also to single-layer and multi-layer index systems. In order to mine the fuzziness of data system, Mahalanobis distance and fuzzy neighborhood relationship are employed to generate the fuzzy similarities among samples and fuzzy memberships between the samples induced by conditional attributes and decision classes. Therefore, the Bayes risk model characterized by the aforementioned fuzziness can be called as a fuzzy Bayes risk model. The loss function in Bayes risk, however, is usually determined by experts or through a large number of statistical tests, which greatly limits the practical application and extension of Bayes risk theory [36]. In order to cope with this drawback, a novel loss function model based on Gaussian kernel is proposed. Furthermore, an improved loss function model combined with Gaussian kernel and Mahalanobis distance is designed to determine weights for multi-layer data system. Subsequently, a number of experiments, including the parameter selection tests and comparison experiments, are carried out to illustrate the superiority of the proposed method. Finally, we demonstrate and verify the applicability of the proposed method through the effectiveness evaluation of fighter. Therefore, the main highlights of this work lie in that

1) This paper is the first attempt to deal with the problem of labeled multiple attribute decision making.
2) A simple and effective objective attribute weight assignment method named MGFBRW is proposed.
3) A Gaussian kernel loss function model is raised, which can promote the application and extension of Bayes risk theory.
4) The detailed demonstrations and analyses of fighter effectiveness evaluation have important guiding significance for other similar engineering applications.

[1] In fact, systems with multiple decision attributes can also be transformed into ones with single decision attribute.

The remainder of this paper is organized as follows. Section 2 introduces the LMADM problem for this work. The basic theories and analyses of the proposed method are presented in Section 3. The results and analyses of numerical experiments are given in Section 4, and the effectiveness evaluation of fighter is demonstrated in Section 5. Then, some discussions are brought in Section 6. Finally, conclusions and future work are described in Section 7.

2. LMADM Problem

In this section, the LMADM problem related to our work will be carried out first, then the goal of LMADM is analyzed, which will pave the way for the further development of the following sections.

2.1. Labeled multiple attribute decision making problem

Definition 1 (Decision system)**.** [37] A decision system is a 4-tuple $DS = (U, \{A|A = C \cup D\}, \{V_a|a \in A\}, \{I_a|a \in A\})$, where U is a finite set of objects called universe and $U = \{x_1, x_2, \cdots, x_m\}$, A is the attribute set, C is the set of conditional attributes, D is the decision attribute, $C \cap D = \emptyset$, $D \neq \emptyset$, V_a is a set of values of each $a \in A$, and I_a is an information function for each $a \in A$.

A decision system is often denoted as $DS = (U, A, V, I)$ or $DS = (U, C, D)$ for short. Specifically, a decision system is called an information system $IS = (U, C)$ if its decision attribute forms an empty set [38].

The LMADM problem used in DS is a special case of MADM. The decision attribute (label) provides an initial rough classification label for the whole data system. However, in MADM, we expect to acquire the ordering relationship of each alternative. Therefore, the specific implication and resolution process of LMADM can be described as follows.

Definition 2 (LMADM)**.** Given a decision system $DS = (U, C, D)$, $U = \{x_1, x_2, \cdots, x_m\}$ is the set of alternatives, $C = \{c_1, c_2, \cdots, c_n\}$ is the set of conditional attributes, $D = \{d_1, d_2, \cdots, d_K\}$ $(K \leq m)$ is the set of labels associated with the alternatives, $W = (w_1, w_2, \cdots, w_n)$ generated by some means is the weight vector of C, such that $\sum_{j=1}^{n} w_j = 1$ and $w_j \geq 0$, $V = [v_{ij}]_{m\times n}$ is a decision matrix given by the decision maker, where v_{ij} denotes the preference value of x_i induced by c_j.

It is worth noting that, the DMs often tend to be dishonest in the process of decision making because of their personal preferences [9], which has become a complex and difficult problem in the study of MADM. In this regard, a number of research results have been reported in the literature (see [6–11]). In this paper, in order to simplify the study, we assume that the DMs are honest in MADM and employ the objective weight assignment methods to maximize the possibility of avoiding the participation of DMs. As for the dishonest topic, it is not the focus of this paper and the reader interested in this issue can refer to the references [9,39–42].

Generally, there are four steps in LMADM by using objective weight assignment methods, which can be listed as follows.

1) Normalization
   In order to avoid the dimensional problem interfering with decision making, the raw data system should be normalized, where the cost normalized model (Eq. (1)) and the income normalized model (Eq. (2)) are employed if $c_j \in C$ is a cost and benefit attribute [43], respectively.

$$\bar{v}_{ij} = \frac{\max_j(v_{ij}) - v_{ij}}{\max_j(v_{ij}) - \min_j(v_{ij})}, \tag{1}$$