Knowledge-Based Systems 000 (2018) 1–11

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

An evidential dynamical model to predict the interference effect of categorization on decision making results

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ARTICLE INFO

Article history:
Received 27 August 2017
Revised 3 March 2018
Accepted 8 March 2018
Available online xxx

Keywords:
Evidential dynamical model
Quantum dynamical model
Dempster–Shafer evidence theory
Interference effect of categorization
Disjunction fallacy
Categorization decision-making experiment

ABSTRACT

Categorization is necessary for many decision making tasks. However, the categorization process may interfere the decision making result and bring about the disjunction fallacy. To predict the interference effect of categorization, some models based on quantum cognition theory have been proposed. In quantum dynamical models, like the quantum belief-action entanglement (BAE) model, actions and beliefs are deemed to be entangled. However, the entanglement degree is an artificially defined parameter. In this paper, a new evidential dynamical (ED) model based on Dempster–Shafer (D-S) evidence theory and quantum dynamical modelling is proposed. Considering that sometimes people hesitate to make a decision, it is reasonable to extend the action states by introducing an uncertain state. In an evidential framework, categorization can influence the uncertain state in actions. The interference effect is measured by handling the uncertain state while no extra parameter is defined artificially. The proposed model is applied to the classical categorization decision-making experiments. Compared with the existing models, the number of free parameters in the ED model is less than the classical quantum models, and the ED model is more rational and simpler than an evidential Markov model. The model application results and discussions show the correctness and effectiveness of the ED model. Not only the interference effect of categorization on decision making results is explained and predicted, but also an inspiring dynamical decision making framework is proposed in this paper. We believe that the proposed ED model will bring more opportunities and will result in more applications in the future.

1. Introduction

As an important and normal part of decision making process, categorization is widely involved in decision making tasks in reality [1–3]. Without precise categorization, many decisions can not be made. For instance, doctors must classify the tumor before doing the surgery; judges need to categorize the defendant before making a judgement; the commander need to categorize an unexpected aircraft before making a command. Additionally, categorization is also an important task in knowledge-based systems [4–6], which is the basis of decision making. However, lots of practical examples and experiments show that categorization may result in the disjunction fallacy, which violates the law of total probability [7,8]. In this case, some categorization-based decisions may be counterintuitive, which should be paid attention to.

Townsend et al. [9] proposed a categorization decision-making experiment, which is now a widely used paradigm for studying the disjunction effect of categorization. A Markov process can be used to model a random system that changes states according to a transition rule that only depends on the current state [10,11]. Unfortunately, however, the disjunction effect can not be predicted in a Markov model [9]. To address it, some quantum models have been proposed. Quantum probability theory is an effective tool to explain this phenomenon and predict the interference effect, which has been widely applied in the fields of cognition and decision making [12–14]. It is an effective approach to psychology and behavioral sciences [15–17]. Besides, the quantum framework has been widely applied in wide studies, including conceptual combination [18], cognition [19,20], reliability analysis [21], data fusion [22], optimization [23] and so on. Many paradoxes can be explained in a quantum framework, like the violation of the sure thing principle [24], the additive law of probability [25], the Ellsberg paradox paradox [26], and so on. It can also explain many irregular phenomena in classical theories, like order effect [27–29], disjunction fallacy [30–32], the prison dilemma [33], etc. Among them, the disjunction fallacy is one of the most popular problems. In a quantum model, the interference effect, a term in quantum mechanics, is borrowed to explain the disjunction fallacy. Many theories have been proposed to predict the interference

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https://doi.org/10.1016/j.knosys.2018.03.014
0950-7051/

effect, such as the quantum dynamical (QD) model [7,34,35], the quantum-like model [36–38], the quantum prospect decision theory [39–41] and the quantum-like Bayesian networks [42,43], etc. But most of the existing quantum models encounter a problem of introducing extra quantum-related parameters, which can't be determined clearly enough.

To predict the interference effect of categorization, in our previous work, we modeled the decision making process under an uncertain environment [44]. Decision making and optimization under uncertainty is normal in reality and has been heavily studied [45–47]. But it is still an open issue for uncertain information modeling and processing [48–50]. Many theories have been developed, such as fuzzy set theory [51,52], Dempster–Shafer (D-S) evidence theory [53,54], D number [55,56], Z number [57] and so on. Among them, D-S evidence theory is a powerful tool to handle the uncertainty [58–60]. It has been widely used in many applications, like risk analysis [61,62], pattern recognition [63–65], dependence assessment [66,67], multiple attribute decision making [68–70] and so on. In recent years, D-S theory has been widely combined with quantum theories [71–74]. An evidential Markov (EM) decision making model which combines D-S theory with Markov modelling to predict the disjunction effect was presented in previous work [44]. Nevertheless, the EM model is based on an assumption of holding tendency to certain decision under uncertainty, which is hard to be quantitatively proven.

In this paper, an evidential dynamical (ED) model based on D-S theory and quantum dynamical modelling is proposed to predict the interference effects of categorization on decision making results. Generally, a decision making process consists of a belief part and an action part. Numerous cases show that people may hesitate to make a certain decision during the decision making process. Hence, to address it, the action states are extended in an evidential framework by introducing an uncertain state, while the uncertainty in beliefs is represented as a superposition of certain states. The uncertain action state can be influenced by categorization which happens in beliefs. In a quantum theory, before making a final decision, human thoughts are seen as superposed waves that can interfere with each other. In the classical QD model, the interference process is driven by an extra parameter. In the quantum belief-action entanglement (BAE) model [8], this parameter is named as entanglement degree because the actions and beliefs are deemed to be entangled. The psychological function of entanglement is applied to coordinate beliefs and actions. Its usage is motivated by a need for explaining why people may be inclined to change their beliefs to be consistent with their own actions [75]. To the contrary, in the ED model, although the action states are extended, the number of free parameters decreases as no more parameter is defined artificially. Definitions of the remaining parameters are based on an optimization principle. Then the interference effect can be well predicted and measured by handling the uncertain state in actions. The ED model is applied to classical categorization decision-making experiments. ED model application results and their comparison with results obtained by means of using other approaches show that the model is an effective and efficient tool for predicting of the interference effects of categorization on decision making results.

The rest of the paper is organized as follows. In Section 2, the preliminaries of the basic theory employed will be briefly introduced. The background of the categorization decision-making experiment is illustrated in Section 3. Then our ED model is proposed to predict the interference effect and explain the experiment results in Section 4. The results of parameter determination, model application and sensitivity analysis are shown in Section 5. The ED model is compared with other three models in Section 6. Finally, Section 7 comes to the conclusion.

2. Preliminaries

2.1. Quantum dynamical model

The QD model first proposed by Busemeyer et al. in 2006 [35] is formulated as a random walk decision process. The evolution of complex valued probability amplitudes over time is described. The interference effect can be produced in a quantum model which is not possible in a classical Markov model. The QD model assumes that a participant has some potential to be in every state in the beginning. Thus the state is a superposition of all possible n states

$$|\psi\rangle = \psi_1|S_1\rangle + \psi_2|S_2\rangle + \cdots + \psi_n|S_n\rangle, \tag{1}$$

and the initial state corresponds to an amplitude distribution $\psi(0)$ represented by the $n \times 1$ matrix

$$\psi(0) = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{bmatrix}. \tag{2}$$

During the decision making process, the state evolves across time obeying a Schr ö dinger equation:

$$\frac{d}{dt}\psi(t) = -i \cdot H \cdot \psi(t), \tag{3}$$

where H is a Hermitian matrix: $H^\dagger = H$, which has elements h_{ij} in row i and column j representing the instantaneous rate of change to $|i\rangle$ from $|j\rangle$. Eq. (3) has a matrix exponential solution:

$$\psi(t_2) = e^{-iHt} \cdot \psi(t_1) = U(t) \cdot \psi(t_1), \tag{4}$$

where matrix $U(t) = e^{-iHt}$ is a unitary matrix, which satisfies: $U(t)^\dagger U(t) = I$. It finally guarantees that $\psi(t)$ always has unit length. For $t = t_2 - t_1$, the transition probability is determined as:

$$T(t) = |U(t)|^2. \tag{5}$$

The element T_{ij} represents the probability of observing state i at time t_2 given that state j was observed at time t_1. Based on the above definition, the amplitude distribution of state evolves to $\psi(t)$ from the initial $\psi(0)$ across time t as Eq. (6):

$$\psi(t) = U(t) \cdot \psi(0), \tag{6}$$

which shows the dynamics in a decision making process.

2.2. Dempster–Shafer evidence theory

In Dempster–Shafer evidence theory, frame of discernment F is a fixed set of N mutually exclusive and exhaustive elements, symbolized by $\Theta = \{H_1, H_2, \ldots, H_N\}$. Let us denote $P(\Theta)$ as the power set composed of 2^N elements A of Θ:

$$P(\Theta) = \{\emptyset, \{H_1\}, \{H_2\}, \ldots, \{H_N\}, \{H_1 \cup H_2\}, \{H_1 \cup H_3\}, \ldots, \Theta\}$$

The support degree of an element is described by a mass function, which is defined as a mapping m from the power set to [0, 1], satisfying:

$$\sum_{A \subseteq P(\Theta)} m(A) = 1,$$
$$m(\emptyset) = 0, \tag{7}$$

where A is a subset of $P(\Theta)$, called the focal set. The mass function is also named as basic probability assignment (BPA) or basic belief assignment (BBA). From a mass function m, we can compute a belief function and a plausibility function, defined as

$$Bel(A) = \sum_{B \subseteq A} m(B)\ , A \subseteq P(\Theta) \tag{8}$$