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# Personalized individual semantics based on consistency in hesitant linguistic group decision making with comparative linguistic expressions

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ABSTRACT

In decision making problems, decision makers may prefer to use more flexible linguistic expressions instead of using only one linguistic term to express their preferences. The recent proposals of hesitant fuzzy linguistic terms sets (HFLTSs) are developed to support the elicitation of comparative linguistic expressions in hesitant decision situations. In group decision making (GDM), the statement that words mean different things for different people has been highlighted and it is natural that a word should be defined by individual semantics described by different numerical values. Considering this statement in hesitant linguistic decision making, the aim of this paper is to personalize individual semantics in the hesitant GDM with comparative linguistic expressions to show the individual difference in understanding the meaning of words. In our study, the personalized individual semantics are carried out by the fuzzy envelopes of HFLTSs based on the personalized numerical scales of linguistic term set.



## 1. Introduction

In real-world decision making, Computing with Words (CW) is often applied as a basis to solve the decision problems with linguistic information [14–16,28,29]. In recent years, different linguistic models are proposed for CW. Particularly, the 2-tuple linguistic representation model [8] provided a computation technique to deal with linguistic information without loss of information. Based on the 2-tuple linguistic representation model, the model based on a linguistic hierarchy [6] and the numerical scale model [2,3] are developed to provide good methods to deal with the linguistic decision making problems with single linguistic term.

However, the complexity and time pressure of decision making problems nowadays make decision makers need more elaborated expressions than a simple linguistic label [20]. Hence, to overcome this limitation, Rodríguez et al. [21] introduced the concept of hesitant fuzzy linguistic term set (HFLTS) to serve as the basis of increasing the flexibility of the elicitation of linguistic information by means of linguistic expressions.

To generate more elaborate linguistic expressions, Rodríguez et al. [21] provided a method to generate comparative linguistic expressions by using a context-free grammar and HFLTS. To deal with comparative linguistic expressions in group decision making (GDM), a decision model was proposed in [22] to facilitate the elicitation of linguistic information in hesitant situation. Besides, to represent the semantics of comparative linguistic expressions, Liu and Rodríguez [11] proposed a representation way by means of a fuzzy envelope to carry out the CW processes and discussed its application in multicriteria decision making. Some further developments about the hesitant linguistic decision making can be found in [19,23].

In GDM dealing with CW, there is a fact that words mean different things for different people [5,15,16]. For example, when evaluating the quality of a paper, three reviewers think the paper has “good” quality, but this term “good” has different semantics for these three reviewers. That makes the understanding and numerical meanings of “good” for different reviewers are different. The existing studies use the type-2 fuzzy sets [15] and multi-granular linguistic models [7,17] for managing this issue. Although both methods deal with multiple meanings of words are quite useful,

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they do not represent yet the specific semantics of each individual. To overcome this problem, Li et al. [10] proposed a personalized individual semantics approach to model and solve linguistic GDM by means of numerical scales [1–3] and the 2-tuple linguistic model [8] to improve the management of different meanings of words for different people. This approach shows the good features for managing linguistic information in CW processes and can reflect individual personalized differences in understanding the meaning of words.

In hesitant linguistic decision making, although there are many studies (e.g., [2,11,15,26]) to discuss the representations of HFLTSs, few studies consider the personalized individual semantics among decision makers when expressing the preferences using HFLTSs. Therefore, in this paper, we apply the idea of personalize individual semantics to reflect the different understanding of words for different decision makers in hesitant linguistic decision making. A new framework to personalize individual semantics in hesitant linguistic GDM with comparative linguistic expressions is proposed. This proposal consists of a two-step procedure:

- An average consistency-driven model is proposed to set personalized numerical scales for linguistic terms with comparative linguistic expressions. The proposed model is based on measuring the average consistency index (ACI) of hesitant fuzzy linguistic preference relations (HFLPRs) and provides a basis for developing the personalized individual semantics of HFLTSs.
- Based on the personalized numerical scales obtained from the average consistency-driven model, a process to personalize individual semantics with comparative linguistic expressions via the fuzzy envelope for HFLTSs represented by fuzzy membership function is proposed.

The proposed personalized individual semantics show the individual difference in understanding the meaning of comparative linguistic expressions. The use of the personalized individual semantics provides a new way to show decision makers' numerical meaning individually, and also provides a potential tool to obtain the optimal solution in hesitant linguistic GDM when dealing with the fact that words mean different things to different people.

The rest of this paper is arranged as follows. In Section 2, we present some basic knowledge. Then, in Section 3 the framework and models to personalize individual semantics with comparative linguistic expressions are proposed. Next, Section 4 provides numerical examples and analysis. Section 5 discusses the advantages and weakness of the proposed model. Section 6 concludes this paper with final remarks.

## 2. Preliminaries

In this section, we introduce the basic knowledge regarding the 2-tuple linguistic model, numerical scale, comparative linguistic expressions and HFLTSs.

### 2.1. The 2-tuple linguistic model and numerical scale

The 2-tuple linguistic representation model, presented by Herrera and Martínez [8], represents the linguistic information by a 2-tuple $(s_i, \alpha) \in \bar{S} = S \times [-0.5, 0.5)$, where $s_i \in S$ and $\alpha \in [-0.5, 0.5)$.

**Definition 1** [8]. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ be a value representing the result of a symbolic aggregation operation. The 2-tuple linguistic value that expresses the equivalent information to $\beta$ is then obtained as:

$\Delta : [0, g] \to \bar{S}$,

being

$$\Delta(\beta) = (s_i, \alpha), \quad with \begin{cases} s_i, & i = round(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5) \end{cases}$$

Function $\Delta$, it is a one to one mapping whose inverse function $\Delta^{-1} : \bar{S} \to [0, g]$ is defined as $\Delta^{-1}(s_i, \alpha) = i + \alpha$. When $\alpha = 0$ in $(s_i, \alpha)$ is then called simple term.

A computational model for the 2-tuple linguistic model was defined in [8], in which different operations were introduced:

(1) A 2-tuple comparison operator: Let $(s_k, \alpha)$ and $(s_l, \gamma)$ be two 2-tuples. Then:
   (i) if $k < l$, then $(s_k, \alpha)$ is smaller than $(s_l, \gamma)$.
   (ii) if $k = l$, then
      (a) if $\alpha = \gamma$, then $(s_k, \alpha)$, $(s_l, \gamma)$ represents the same information.
      (b) if $\alpha < \gamma$, then $(s_k, \alpha)$ is smaller than $(s_l, \gamma)$.

(2) A 2-tuple negation operator:
$Neg((s_i, \alpha)) = \Delta(g - (\Delta^{-1}(s_i, \alpha)))$.

(3) Several 2-tuple aggregation operators have been developed (see [8,14]).

The concept of the numerical scale was defined to transform linguistic terms into real numbers:

**Definition 2** [3]. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set, and $R$ be the set of real numbers. The function: $NS$: $S \to R$ is defined as a numerical scale of $S$, and $NS(s_i)$ is called the numerical index of $s_i$. If the function $NS$ is strictly monotone increasing, then $NS$ is called an ordered numerical scale.

Based on the concept of numerical scale, Dong et al. [2] proposed a connection of the numerical scale model with the 2-tuple linguistic model [8], the proportional 2-tuple linguistic model [25] and the model based on a linguistic hierarchy [6], respectively, by setting different certain values for $NS(s_i)$.

### 2.2. Comparative linguistic expressions and HFLTSs

To facilitate the elicitation of flexible and rich linguistic expressions, Rodríguez et al. [21] proposed an approach to generate comparative linguistic expressions by using a context-free grammar.

**Definition 3** [21]. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and $G_H$ be a context-free grammar. The elements of $G_H = \{V_N, V_T, I, P\}$ are defined as follows,

$V_N = \{\langle primary\ term\rangle, \langle composite\ term\rangle, \langle unary\ relation\rangle, \langle binary\ relation\rangle, \langle conjunctiond\rangle\}$

$V_T = \{lower\ than, greater\ than, between, and, s_0, s_1, \ldots, s_g\}$

$I \in V_N$.

For the context-free grammar $G_H$, the production rules are as follows:

$P = \{I ::= \langle primary\ term\rangle | \langle composite\ term\rangle\ \langle composite\ term\rangle$

$::= \langle unary\ relation\rangle \langle primary\ term\rangle | \langle binary\ relation\rangle \langle primary\ term\rangle$

$\langle conjuction\rangle \langle primary\ term\rangle$

$\langle primary\ term\rangle ::= s_0 | s_1 | \ldots | s_g$

$\langle unary\ relation\rangle ::= lower\ than | greater\ than$

$\langle binary\ relation\rangle ::= between$

$\langle conjuction\rangle ::= and\}$

By using the context-free grammar $G_H$, the comparative linguistic expressions are generated. Since they cannot be directly used for CW, Rodríguez et al. [21] provided a transformation function to transform them into HFLTSs.

**Definition 4** [21]. Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set. A HFLTS, $H_S$, is an ordered finite subset of consecutive linguistic terms of $S$.