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An uncertain single machine scheduling problem with periodic maintenance

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ABSTRACT

This paper studies a single machine scheduling problem with periodic maintenance, in which processing time and repair time are nondeterministic. In order to deal with nondeterministic phenomena, uncertainty theory is introduced to minimize the makespan under an uncertain environment. Three uncertain programming models are proposed, which can be converted into deterministic forms based on the uncertainty inverse distribution. List scheduling (LS) and longest processing time (LPT) algorithms are employed to solve the problem. It is proved that the two algorithms have the same worst cast ratio under different confidence levels and the LPT algorithm has a better performance bound. A hybrid intelligent algorithm for the problem is designed and some numerical experiments demonstrate the effectiveness of the proposed models and algorithm.

1. Introduction

In most literatures it is assumed that all machines are available at all time. However, in a real manufacturing environment, the machines could be unavailable due to many reasons. In order to improve longevities, performances and efficiency of machines, the preventive maintenance is treated as a basic method. The machine maintenance is often regarded as an unavailability constraint in some literatures. Lee [10] considered various kinds of scheduling problems where one of these machines is always available and other machines have one unavailable period. Each case was discussed separately. He et al. [7] considered a single machine scheduling problem with rate-modifying activity and they proposed a pseudo-polynomial time optimal algorithm for the problem. Chen [2] addressed a single machine scheduling problem with periodic maintenance where the machine stopped periodically during the rolling horizon, proposing a heuristic algorithm to deal with the problem successfully. Ji et al. [8] considered an availability constraint on a single machine scheduling problem with linear deteriorating processing time. Tan et al. [22] proved shortest processing time algorithm has a worst-case ratio of $\frac{3}{2}$ and 2 respectively on two parallel identical machines with given unavailable periods. Liu et al. [15] considered two online scheduling problems with periodic availability constraint. Zhong et al. [30] studied an order acceptance scheduling model with machine availability constraint and discussed the approximability of the model. Luo et al. [17] considered a variable maintenance activity on a single scheduling machine and polynomial time algorithms for different criteria are proposed. Sbihi and Varnier [20] considered a single machine scheduling problem with several maintenance periods. Two cases about the maintenance period were investigated and an efficient heuristic for these problems was proposed.

Almost all studies on machine maintenance are assumed to be in a deterministic environment. However, human behaviors are involved in job processing and machine maintenance according to the real situation, thus a lot of man-made factors, environmental factors and the machine structure have a potential influence on scheduling horizon. As it is known, probability theory has been widely used to deal with indeterminacy factors for a long time. However, it is unreasonable to deal with all indeterminacy factors by means of probability theory. The premise of applying probability theory is that the probability distribution is close to the cumulative frequency. In many scheduling problems, the probability distribution is hard to obtain due to lack of accurate data and in this case, it is the only way to invite experts to evaluate the belief degree that an uncertain event will occur and the degree of faith depends largely on personal knowledge. In order to deal with the involved human uncertainty, uncertainty theory was founded by Liu [12] in 2007 and refined it in 2010 [13]. Uncertainty theory is a branch of axiomatic mathematics for modeling human uncertainty, which has been deeply developed in many fields such as

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uncertain programming [4,9,19,21,23,28,29], uncertain risk analysis [11,16,18], and uncertain uncertain calculus [3,25–27].

In order to understand uncertainty theory better, an example of uncertain variables is given. Consider a bridge and its strength. At first, we have to admit that no destructive experiment is allowed for the bridge. Thus we have no samples about the bridge strength. In this case, there do not exist any statistical methods to estimate its probability distribution. How do we deal with it? It seems that we have no choice but to invite some bridge engineers to evaluate the belief degrees about the bridge strength. In fact, it is almost impossible for the bridge engineers to give a perfect description of the belief degrees of all possible events. Instead, they can only provide some subjective judgments about the bridge strength. As a simple example, we assume a consultation process is as follows:

(Q) What do you think is the bridge strength?

(A) I think the bridge strength is between 80 and 120 tons.

What belief degrees can we derive from the answer of the bridge engineer? First, we may have an inference:

(i) I am 100% sure that the bridge strength is less than 120 tons.

This means the belief degree of "the bridge strength being less than 120 tons" is 1. Thus we have an expert' s experimental data (120, 1). Furthermore, we may have another inference:

(ii) I am 100% sure that the bridge strength is greater than 80 tons.

This statement gives a belief degree that the bridge strength falls into the right side of 80 tons. We need translate it to a statement about the belief degree that the bridge strength falls into the left side of 80 tons:

(ii') I am 0% sure that the bridge strength is less than 80 tons. Although the statement (ii') sounds strange to us, it is indeed equivalent to the statement (ii). Thus we have another expert' s experimental data (80, 0).

Until now we have acquired two expert' s experimental data (80, 0) and (120, 1) about the bridge strength. Could we infer the belief degree $\Phi(x)$ that the bridge strength falls into the left side of the point x? The answer is affirmative. For example, a reasonable value is

$$\Phi(x)=\begin{cases}0, & \text{if } x<80\\ (x-80)/40, & \text{if } 80\le x\le 120\\ 1, & \text{if } x>120.\end{cases} \tag{1}$$

From the function $\Phi(x)$, we may infer that the belief degree of "the bridge strength being less than 90 tons" is 0.25. In other words, it is reasonable to infer that "I am 25% sure that the bridge strength is less than 90 tons", or equivalently "I am 75% sure that the bridge strength is greater than 90 tons".

In this paper, a single machine scheduling problem with periodic maintenance is studied. Under the affection from many parts, it doesn't seem reasonable to consider all kinds of scheduling problems in a deterministic environment, especially for those with machine maintenance or fault. In order to grasp the real-time situation and make accurate instructions, decision makers are more likely to consider more practical factors. Therefore, it is of great theoretical significance and practical value to analyze the problem in an uncertain environment. The objective is to minimize the makespan under this circumstance. Moreover, for small scale problems, expected value model, pessimistic value model, and measure chance model are proposed. In fact, the proposed models can be converted into deterministic forms. Based on prior theoretical analysis, an efficient hybrid intelligent algorithm is proposed.

The rest of the paper is organized as follows. In Section 2, some basic definitions about uncertainty theory are introduced. In Section 3, three models under uncertain environment are constructed and the equivalent forms of models are obtained. In Section 4, worst case ratios of LS algorithm and LPT algorithm for the problem are analyzed. Two practical experiments in Section 5 are presented to verify the effectiveness the proposed algorithms. In Section 6, a hybrid intelligent algorithm is proposed based on the previous theoretical analyses. Numerical experiments are implemented to illustrate the validity of the proposed models and approach in the Section 7.

2. Preliminary

Uncertainty theory founded by Liu [12] in 2007 and refined by Liu [13] in 2010, is a branch of axiomatic mathematics for modeling human uncertainty. Let Γ be a nonempty set, Γ a σ-algebra over Γ, and each element Λ in $\mathcal{L}$ is called an event. Uncertain measure is defined as a function from $\mathcal{L}$ to [0,1]. In detail, Liu [12] gave the concept of uncertain measure as follows:

Definition 1 [12]**.** Let Γ be a nonempty set, and $\mathcal{L}$ a σ-algebra over Γ. Each element $\Lambda\in\mathcal{L}$ is called an event. A set function $\mathcal{M}$ from $\mathcal{L}$ to [0, 1] is called an uncertain measure if it satisfies the following axioms:

Axiom I. $\mathcal{M}\{\Gamma\}=1$ for the universal set Γ;

Axiom II. $\mathcal{M}\{\Lambda\}+\mathcal{M}\{\Lambda^c\}=1$ for any event Λ;

Axiom III. $\mathcal{M}\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\}\le\sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}$ for every countable sequence of events $\Lambda_1,\Lambda_2,\cdots$.

Besides, the product uncertain measure on the product σ-algebra $\mathcal{L}$ was defined by Liu [14] as follows: as: Let $(\Gamma_k,\mathcal{L}_k,\mathcal{M}_k)$ be uncertainty spaces for $k=1,2,\cdots$. The product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying $\mathcal{M}\left\{\prod_{i=1}^{\infty}\Lambda_k\right\}=\bigwedge_{i=1}^{\infty}\mathcal{M}_k\{\Lambda_k\}$, where Λ_k are arbitrarily chosen events from $\mathcal{L}_k$ for $k=1,2,\cdots$, respectively.

Theorem 1 [13]**.** *Uncertain measure $\mathcal{M}$ is a monotone increasing set function. That is, for any events $\Lambda_1\subset\Lambda_2$, we have*

$$\mathcal{M}\{\Lambda_1\}\le\mathcal{M}\{\Lambda_2\}.$$

Theorem 2 [13]**.** *Suppose that $\mathcal{M}$ is an uncertain measure. Then for any events Λ_1 and Λ_2, we have*

$$\mathcal{M}\{\Lambda_1\}+\mathcal{M}\{\Lambda_2\}-1\le\mathcal{M}\{\Lambda_1\cap\Lambda_2\}\le\mathcal{M}\{\Lambda_1\}\wedge\mathcal{M}\{\Lambda_2\}.$$

An uncertain variable is a measurable function ξ from an uncertainty space $(\Gamma,\mathcal{L},\mathcal{M})$ to the set R of real numbers, i.e., for any Borel set B of real numbers, the set $\{\xi\in B\}=\{\gamma\in|\xi(\gamma)\in B\}$ is an event. The uncertain distribution Φ of an uncertain variable ξ is defined by $\Phi(x)=\mathcal{M}\{\xi\le x\}$ for any real number x.

Theorem 3 [14]**.** *Let ξ be an uncertain variable with continuous uncertainty distribution Φ. Then for any real number x, we have*

$$\mathcal{M}\{\xi\le x\}=\Phi(x),\quad \mathcal{M}\{\xi\ge x\}=1-\Phi(x).$$

Example 1. Linear uncertain variable $\xi\sim\mathcal{L}(a,b)$ has an uncertainty distribution

$$\Phi(x)=\begin{cases}0, & \text{if } x\le a\\ \dfrac{x-a}{b-a}, & \text{if } a\le x\le b\\ 1, & \text{if } x\ge b\end{cases}$$

where a and b are real numbers with $a<b$.

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