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Three-way decisions in ordered decision system

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ABSTRACT

As a natural extension of three-way decisions, this paper presents a novel three-way decision model with order information. First, we do some comparative analysis between ordered three-way decisions and decision-theoretic rough sets, and then present some important properties of the proposed model. Second, a hybrid decision table consisted both of the "order information" and "loss function", is utilized to solve the ordered three-way decisions with two classification problem. Two order sets (dominating set and dominated set generated by order relation) and three risk strategies (optimistic strategy, equable strategy, pessimistic strategy) are induced to construct the model and design the algorithm of ordered three-way decisions. At last, an illustrative example of salary administration validates the reasonability and effectiveness of our approach.

1. Introduction

Three-way decisions (3WD), as a novel granular computing methodology to deal with uncertainty decision problems, has been developed very rapidly since it was proposed by Yao in 2010 [60,61]. It utilizes a risk minimization perspective to illustrate the three types of decision rules which generated by the three regions (positive, negative and boundary regions) in rough sets. Under the Bayesian risk decision criterion, the positive regions generate positive rules to make a decision of acceptance; the negative regions generate negative rules to make a decision of rejection. In addition, the third choice, generated from the boundary regions, makes a decision of noncommitment or deferment [64]. The essential ideas of three-way decisions have been successfully applied in different domains, such as management decisions [8,32,33,35,45], information systems and engineering [2,7,53,70,73], medical management [46,56], etc.

Inspired by Yao's ideas in [64], the existing literatures on three-way decisions can be divided to two scenarios. The first scenario is the generalized three-way decisions, which focuses on the intension and the extension of a concept in a set-theoretical setting. Yao [64] gave many generalizations of sets to describe three-way decisions (e.g, interval sets and three-valued logic, rough sets, fuzzy sets, three-valued approximations and shadowed sets). Ciucci and Dubois [4,5] investigated the dependencies among three-valued logics. Hu [16] systematically analyzed three-way decision spaces in rough sets. Liu et al. [33] constructed a "four-level" probabilistic model criteria for 3WD. Qi et al. [50] discussed the connections between three-way concept analysis and classical concept analysis. Li et al. [23] researched on three-way cognitive concept learning via multi-granularity. Yao and She [69] analyzed the rough set models in multigranulation spaces. The second scenario is the special three-way decisions, which pays more attention to interpret the semantics of 3WD. The most typical representative model of special three-way decisions is decision-theoretic rough sets (DTRS) [57]. In DTRS, the two thresholds can be directly calculated by minimizing the decision cost with Bayesian theory [34]. As some extended models of DTRS, Abd El-Monsef and Kilany [1] constructed the semilower and semiupper approximations, and proposed a generalization and modification of DTRS model. Azam and Yao [3], Herbert and Yao [14,15] introduced game theory to rough sets, and systematically studied 3WD in game-theoretic rough sets. Li and Zhou [21] proposed a risk three-way decision making model with three different decision preferences. Liu et al. [34,38], Qian et al. [49], and Yang et al. [55] studied the multi-category 3WD, multi-granulation 3WD and multi-agent 3WD, respectively. Furthermore, Liu and Liang proposed a series of uncertainty three-way decision models by considering different uncertainty decision measures, including stochastic 3WD [39], interval 3WD [25] and fuzzy 3WD [24,26–28,36]. Deng and Yao [6] did the research on decision-theoretic three-way approximations of fuzzy sets. Yao and Zhou [67] gave two Bayesian approaches to rough sets. Yu et al. [70,71] did a series of studies on 3WD methods with clustering analysis. In addition, many scholars did lots of work on the attribute reduction of 3WD, such as positive region based reduction

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and minimum cost based reduction. As to positive region based reduction, Yao and Zhao [59] analyzed various criteria (e.g., decision-monotocity, generality and cost) for attribute reduction in 3WD. Li et al. [22] investigated the monotonicity of positive region in DTRS model, and further considered the decision risk of attribute reduction in DTRS model. As to minimum cost based reduction, Jia et al. [19,20] proposed a cost-based optimal reduct method of 3WD, and then summarized the generalized attribute reduct methods in rough set theory. Min et al. [44] presented a test-cost-sensitive attribute reduction for 3WD by considering on the test cost. Ma et al. [43] investigated a decision region distribution preservation reduction in 3WD model. Liang et al. [29] introduced group decision making (GDM) into 3WD and proposed GDM-based three-way decisions. Sun et al. [52] further studied the three-way group decision making based on multigranulation fuzzy DTRS. Zhang and Yao [72] presented a Gini objective functions for three-way classifications. All the above stated researches make significant contributions on both theories and methodologies for developing 3WD.

However, the aforementioned literatures rarely consider the order information in 3WD. In some real-life applications, the attributes in an information system may have preference-ordered domains (scales), e.g., product quality, market share, and debt ratio [11]. Iwinski [18] investigated the order information system and examined canonical representation of a set of weak orders. Sai et al. [51] did the researches on the data analysis and mining in ordered information table. As a special definition of ordering values of attributions in rough sets, Greco et al. [9] considered the dominance principles in information system and proposed the "Dominance-based Rough Set Approach (DRSA)". They further introduced the consistency level into DRSA and presented "Variable Consistency Dominance-based Rough Set Approach (VC-DRSA)" [10]. Inuiguchi et al. [42] proposed "Variable-Precision Dominance-based Rough Set Approach (VP-DRSA)" to treat errors and missing condition attributes in the framework of DRSA. Greco et al. [12] presented a Bayesian decision theory for dominance-based rough set approach. Yang et al. [54] investigated the knowledge reductions in incomplete ordered information system. Qian et al. [48] discussed interval ordered decision tables and dominance rules. Liu and Li [40] introduced the strict dominance relation and proposed "Strict-Dominance-based Probabilistic Rough Set Approach (SD-PRSA)". Hu [17] considered three-way decision spaces based on partially ordered sets. Yao et al. [65] pointed out three fundamental issues of probabilistic dominance-bases rough sets.

As we stated above, on the one hand, 3WD mainly focus on the loss function in "cost table"; on the other hand, DRSA mainly focus on the order relation in "information table". The connections between "cost table" and "information table" are seldom investigated for the information system with order relation. Observed by this phenomenon, this paper attempts to introduce order information into DTRS, utilizes an integrated ordered decision table, to analyze the three-way decision approach with ordered decision systems (ODS). In particularly, we discuss the two-class classification problem in this paper.

The remainder of this paper is organized as follows. In Section 2, we review some basic concepts of order information, DTRS, and 3WD system in rough sets. In Section 3, a novel three-way decision model with ordered information is proposed. A hybrid ordered decision table, including both of the "order information" and "loss function", is utilized to deal with the ordered three-way decision model. Then, two order relations and three risk strategies are induced to 3WD, the key steps and the algorithm for deriving three-way decisions in ODS are designed. A case study of salary administration is given to illustrate our approach in Section 4. Section 5 concludes the paper and elaborates on future studies.

2. Preliminaries

The basic concepts, notations and results of ordered information systems [9–11,13,40,47,65], and three-way decisions [31,33,37,57,58,60–64,66,74] are briefly reviewed in this section.

2.1. Ordered information systems

Definition 1. An ordered information system is defined as a 4-tuple $OIS=(U,At,V,f)$, where $U=\{x_1,x_2,\cdots,x_m\}$ is a non-empty finite set of objects and $At=\{a_1,a_2,\cdots,a_n\}$ is a finite nonempty set of attributes. $V=\bigcup_{a\in At}V_a$, where V_a is a domain of attribute a. The domain of a condition attribute a is ordered according to a decreasing or increasing preference. $f: U\times At\rightarrow V_a$ is an information function such that $f(x, a)\in V_a$ for every $x\in U$, $a\in At$.

Given an ordered information system $OIS=(U,At,V,f)$. We assume that the domain of a criterion $a\in AT$ is completely preordered by an outranking relation $\succeq$. The condition criterion having a numerical domain, namely, $V_a\subseteq\mathscr{R}$ ($\mathscr{R}$ denotes the set of real numbers). $\forall x,\ y\in U$, $x\succeq_a y$ if "x is at least as good as (outranks) y with respect to criterion a", that is, $x\succeq_a y\Longleftrightarrow f(x, a)\succeq f(y, a)$, and we can define the dominating set $D^+_{\mathcal{P}}(x)$; On the contrary, $\forall x,\ y\in U$, $x\preceq_a y$ if "x is no better than y with respect to criterion a", that is, $x\preceq_a y\Longleftrightarrow f(x, a)\preceq f(y, a)$, and we can define the dominated set $D^-_{\mathcal{P}}(x)$.

An ordered decision system $ODS=(U,C\bigcup CL,V,f)$ is a special case of OIS. If we set $At=C\bigcup CL$ in which $C=\{a_1,a_2,\cdots,a_n\}$ denotes the set of condition attributes and CL denotes the set of decision attributes. In ODS, $\forall x,\ y\in U$, x dominates y with respect to $\mathcal{P}\subseteq C$ (shortly, x $\mathcal{P}$- dominates y) denoted by $xD_{\mathcal{P}}y$, if $f(x, a)\succeq f(y, a)$ for all $a\in C$. $D_{\mathcal{P}}$ is a partial preorder on U, *i.e.*, $D_{\mathcal{P}}$ is a reflexive and transitive binary relation on U. With the definition of $D_{\mathcal{P}}$, we can further define the $\mathcal{P}$-dominating (dominated) set and upward (downward) unions via Definitions 2 and 3, respectively.

Definition 2. Given $\mathcal{P}\subseteq C$ and $x,\ y\in U$, we say that x dominates y with respect to $\mathcal{P}$, denoted by $xD_{\mathcal{P}}y$, if $\forall q\in\mathcal{P}$, $x\succeq_q y$ holds. Let

$$D^+_{\mathcal{P}}(x)=\{y\in U: yD_{\mathcal{P}}x\}=\{y\in U|\forall a\in\mathcal{P},\ f(y,a)\succeq f(x,a)\};$$
$$D^-_{\mathcal{P}}(x)=\{y\in U: xD_{\mathcal{P}}y\}=\{y\in U|\forall a\in\mathcal{P},\ f(y,a)\preceq f(x,a)\}. \quad (1)$$

represent $\mathcal{P}$-dominating set and $\mathcal{P}$-dominated set with respect to $x\in U$, respectively.

Definition 3. For any $X\subseteq U$, the upward lower and upper approximations $\underline{apr}^+$ and $\overline{apr}^+$, the downward lower and upper approximations $\underline{apr}^-$ and $\overline{apr}^-$, can be definite respectively, as follows:

$$\underline{apr}^+(X)=\{x\in U|D^+_{\mathcal{P}}(x)\subseteq X\};$$
$$\overline{apr}^+(X)=\{x\in U|D^-_{\mathcal{P}}(x)\cap X\neq\emptyset\};$$
$$\underline{apr}^-(X)=\{x\in U|D^-_{\mathcal{P}}(x)\subseteq X\};$$
$$\overline{apr}^-(X)=\{x\in U|D^+_{\mathcal{P}}(x)\cap X\neq\emptyset\}. \quad (2)$$

Definition 4. Suppose the decision attribute $\{CL\}$ induces a partition of U into l classes. Let $T=\{1,2,\cdots,l\}$ be the domain of decision criterion. V_{CL} consists of n elements and $CL=\{CL_t, t\in T\}$, where $CL_t=\{x\in U: f(x,CL)=t\}$. Each object $x\in U$ is assigned to one and only one class CL_t. The classes are preference-ordered according to an increasing order of class indices, *i.e.*, for $\forall r,\ s\in T$ such that $r>s$, the objects from CL_r are strictly preferred to the objects from CL_s. The upward and downward unions of classes are defined, respectively, as:

$$CL^{\succeq}_t=\bigcup_{s\geq t}CL_s;$$
$$CL^{\preceq}_t=\bigcup_{s\leq t}CL_s;\ t\in T. \quad (3)$$

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