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A new measure of consensus with reciprocal preference relations: The correlation consensus degree

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ABSTRACT

The achievement of a ‘consensual’ solution in a group decision making problem depends on experts’ ideas, principles, knowledge, experience, etc. The measurement of consensus has been widely studied from the point of view of different research areas, and consequently different consensus measures have been formulated, although a common characteristic of most of them is that they are driven by the implementation of either distance or similarity functions. In the present work though, and within the framework of experts’ opinions modelled via reciprocal preference relations, a different approach to the measurement of consensus based on the Pearson correlation coefficient is studied. The new correlation consensus degree measures the concordance between the intensities of preference for pairs of alternatives as expressed by the experts. Although a detailed study of the formal properties of the new correlation consensus degree shows that it verifies important properties that are common either to distance or to similarity functions between intensities of preferences, it is also proved that it is different to traditional consensus measures. In order to emphasise novelty, two applications of the proposed methodology are also included. The first one is used to illustrate the computation process and discussion of the results, while the second one covers a real life application that makes use of data from Clinical Decision-Making.

1. Introduction

Consensus reaching is an important component in decision making processes, and indeed it plays a key role in the resolution process of group decision making problems. One of the most significant current discussion in consensus research concerns the measurement and achievement of consensus from both a theoretical and applied points of view. On the one hand, establishing and characterising different methodologies to measure consensus have been addressed from a Social Choice perspective [1,3,13]. On the other hand, within the Decision Making Theory framework, modelling group decision making problems in order to reach a higher level of cohesiveness has been managed successfully [15,32,34,38,39,65]. Outside of these main areas, it is possible to find other methodologies that use the idea of consensus in different ways to the aforementioned ones, with [41,46] being representative examples of these methodologies.

Despite the productive research on this area, consensus measurement is still an open-ended research question because the methodology to use in each case is an essential component of the problem. Up to now most studies on consensus measurement have focused on the use of distance/similarity function based measures and association measures, respectively. Among the distance functions used, and worth highlighting, are the Kemeny, Mahalanobis, Mannhattan, Jacard, Dice and Cosine distance functions [1,4,6,17,19,29,31]. Association measures are less widely used than distance functions but it is also possible to find the use of some of them such as the Kendall’s coefficient, the Goodman-Kruskal’s index and the Spearman’s coefficient [18,24,35,44,58]. In this paper we focus on establishing a new consensus measure following the tradition of association measures. Our proposal is based on the original statistical correlation concept, the *Pearson correlation coefficient*. Therefore, this new measure is an alternative to the use of the aforementioned approaches. The Pearson correlation coefficient plays an important role in Statistics and Data Analysis and it is extensively used as a measure of the degree of linear dependence between two variables. It is easy to interpret as well as invariant to certain changes in the variables [52,55,57]. Specifically, in this paper the notion of dependence among elements from correlation

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coefficient as a measure of the cohesiveness between opinions is adopted. This seems natural because the measurement of consensus resembles the notion of a “measure of statistical correlation”, in the sense that the maximum value 1 captures the notion of unanimity as a perfect relationship among agents' preferences (experts' preferences follow the same direction), while the minimum value -1 captures the notion of total disagreement (experts' preferences present a negative relationship). Furthermore, the higher the cohesiveness between experts' preferences, the more positive correlated the preferences are. Similarly, the lower the cohesiveness between experts' preferences, the more negative correlated the preferences are.

This new consensus measure will be developed within assumptions of experts' opinions or preferences being expressed by means of reciprocal preference relations, a framework that is currently of interest to the research community in decision theory under uncertainty [7,27,28,45]. Under reciprocal preference relations, on the one hand and as it was mentioned above, the new proposed approach inherits advantages of previous approaches based on traditional distance/similarity and association measures. On the other hand, maximum consensus traditionally represents the case when experts provide the same preference intensities for each possible pair of alternatives. This, though, is not the only possible scenario of maximum consensus. Indeed, the proposal here put forward addresses this issue satisfactorily because maximum possible cohesiveness or consensus between experts' opinions does not necessary imply that all reciprocal preference relations have to coincide, and therefore all experts do not necessary need to have the same preference intensities in all possible pairs of alternatives. It is sufficient, though, that experts rank alternatives in the same way. To support all these claims, a set of properties verified by the new proposed measure of consensus, the *correlation consensus degree*, are proved. These properties ensure the suitability of the correlation consensus degree. Furthermore, in order to emphasise novelty, two applications of the proposed methodology are also included. The first one is used to illustrate the computation process and discussion of the results, while the second one covers a real life application that makes use of data from Clinical Decision-Making.

The rest of the paper is organised as follows. Section 2 contains a brief overview of the different approaches in literature to measure group cohesiveness. The basic notation and preliminaries are presented in Section 3. Section 4 provides the new approach to consensus measurement based on the Pearson correlation coefficient. In Section 5, properties of the new correlation consensus degree are studied. Section 6 presents two practical applications of the proposed methodology. Finally, some concluding remarks and future research are presented in Section 7.

2. Consensus measurement in the literature

A considerable amount of literature has been published on measuring and reaching consensus in group decision making problems. Consensus measurement is a prominent and active research subject in several areas such as Social Choice Theory and Decision Making Theory. A brief overview of how this issue has been addressed in recent literature from the aforementioned research areas is provided.

From the Social Choice Theory, the first serious discussions and analysis of consensus measurement from an Arrovian perspective emerged with Bosch's PhD Thesis [13], where both absolute and intrinsic measures of consensus were proposed, analysed and axiomatically characterised. From the point of view of considering consensus among a family of voters, McMorris and Powers [48] characterised consensus rules defined on hierarchies, while García-Lapresta and Pérez-Román [29] focused on how to measure consensus using complete preorders on alternatives and introduced a class of consensus measures based on seven well-known distances. Subsequently, Alcalde-Unzu and Vorstatz in [1] characterised a family of linear and additive consensus measures, whereas in [2] new ways to measure the similarity of preferences in a group of individuals were suggested. Alcantud, de Andrés Calle and Cascón [3] studied and characterised a class of consensus measure, called *referenced consensus measure*, that permits to produce a numerical social evaluation from purely ordinal individual information. This measure has to be specified by means of a voting mechanism and a measure of agreement between profiles of orderings and individual orderings. Moreover, Alcantud, de Andrés Calle and Cascón in [5] contributed to the formal and computational analysis of the aforementioned referenced consensus measure by focusing on two relevant and specific cases: the Borda and the Copeland rules under a Kemeny-type measure. There are, however, situations where each member of a population classifies a list of options as either acceptable or non-acceptable; either agree or disagree, etc., and therefore generating a dichotomous preference structure. Under this assumption, Alcantud, de Andrés Calle and Cascón [4] proposed the concept of *approval consensus measure* and gave axiomatic characterisations of two generic classes of such approval consensus measures. Alcantud, de Andrés Calle and González-Arteaga [6] introduced the use of the Mahalanobis distance for the analysis of the cohesiveness of a group of complete preorders and proved that arbitrary codifications of the preferences are incompatible with their formulation although affine transformations permit to compare profiles on the basis of such proposal. Finally, it is worth mentioning a distance-based approach to measure the degree of consensus considering approval information about alternatives as well as the rankings of them suggested by Erdamar et al. in [25].

From the Decision Making Theory, a considerable amount of contributions have been made since the 1980's. As such, it is worth mentioning the first preliminary work on reaching consensus and its measurements carried out by Kacprzyk and Fedrizzi [42], in which the concept of “degree of consensus” in the sense of expressing the degree to which “most of” the individuals in a group agree to “almost all of” the options. The point of departure of this paper being that the experts' opinions are expressed by fuzzy preference relations. Within this framework of preference representation, different consensus measurement based on similarity measures have been put forward by Herrera-Viedma, et al. [37] and Wu and Chiclana [63] for both complete and incomplete information environments. The case when experts' opinions are expressed by means of linguistic assessments has been extensively studied and it is worth mentioning the works of Ben-Arieh and Chen [12], Cabrerizo, Alonso and Herrera-Viedma [14], García-Lapresta, Pérez-Román [30], Herrera, Herrera-Viedma and Verdegay [36], Herrera-Viedma, et al. [40], Pérez-Asurmendi and Chiclana [53] and Wu, Chiclana and Herrera-Viedma [65]. Finally, models to reach consensus where experts assess their preferences using different preference representation structures (preference orderings, utility functions, multiplicative preference relations and fuzzy preference relations) have also been studied and proposed by Dong and Zhang [23], Fedrizzi et al. [26] and Herrera-Viedma, Herrera and Chiclana [39]. The problem of measuring and reaching consensus with intuitionistic fuzzy preference relations and triangular fuzzy complementary preference relations have also been covered in detail by Wu and Chiclana in [62,64].

To conclude, Table 1 summarises and classifies the approaches that have been reviewed in this Section.

3. Preliminaries

This Section briefly presents the main concepts needed to make the paper self-contained, and as such a short review of