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An intuitionistic fuzzy graded covering rough set

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ABSTRACT

Exploring rough sets from the perspective of covering represents a promising direction in rough set theory, where concepts are approximated by substituting of an equivalent relation in classical rough set theory with a covering in covering-based rough set theory. By combining intuitionistic fuzzy (IF) β-neighborhoods induced by an IF β-covering with IF rough sets, this study develops a new rough set model, which is a generalization of the β-neighborhood fuzzy covering rough sets and IF rough sets. First, we present the concepts of IF graded covering and IF graded neighborhood, namely, IF β-covering and IF β-neighborhood, respectively. We propose an IF graded approximation space on the basis of IF graded neighborhood and discuss its uncertainty measures, namely, information entropy and rough entropy. Second, we generalize an IF rough set based on IF relation to one based on the presented IF graded neighborhood and use the distance between two IF sets to characterize the latter. Third, we present the matrix computation method for the upper and lower approximations of the presented IF rough sets based on the IF graded neighborhood. Fourth, from the multi-granulation perspective, we examine the IF graded approximation space, uncertainty measures, IF rough sets, and computation methods for its reducts. Finally, we discuss several generalizations similar to the presented IF covering rough sets.

1. Introduction

Rough set (RS) theory was introduced by Pawlak [37] as a valid means of knowledge discovery and information processing, in which the fundamental tools consist of relations that represent information systems or decision tables. In rough set theory, two main factors affect the description capability of information systems or decision tables: set approximation and knowledge reduction. On the one hand, given a subset in the universe, two definable sets called lower and upper approximations are induced to approximate the subset. On the other hand, under the condition of keeping the set approximations unchanged, knowledge reduction is conducted to remove the redundant attributes from the information system or decision table to acquire some simpler rules than the original information system or decision table.

Pawlak's classical rough set theory can be employed to handle symbolic data that generate equivalence classes. However, in many real-world situations, attributes of multiple different types exist in information systems or decision tables. On this condition, Pawlak's classical rough set theory has a theoretical limitation on in addressing such information systems. An important method to overcome this limitation is to extend partitions to coverings. As the generalizations of classical rough sets, all types of covering-based rough sets have attracted increasing attention [34,46,48,55,56,58,67], especially in the construction of set approximations [31,32,45,51,65]. Yao and Yao [64] unified and classified various types of set approximations in covering-based rough set theory. Zhu and Wang [77,78] focused on the aspect of the knowledge reduction of covering-based rough sets, and proposed a reduction method to reduce the redundant elements in a covering while preserving the set approximations. Chen et al. [8] originally proposed a discernibility matrix for computing the reduct of a covering decision information system. For further improvement, Wang et al. [53] constructed a new discernibility matrix, that greatly reduced the computational complexity of the original discernibility matrix.

Classical rough set theory is used only to describe crisp sets. To describe fuzzy concepts, Dübois and Prade [16] extended the basic idea of rough sets to a new model called fuzzy rough sets. In fuzzy rough sets, a fuzzy similarity relation is employed to describe the degree of similarity between two objects, instead of the equivalence relation used in the classical rough set model. Many extended versions and relative applications have been developed widely apply the fuzzy rough set method [11–13,44,50]. Aiming at

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fuzzy covering rough sets, Li et al. [27] constructed two pairs of generalized lower and upper fuzzy rough approximation operators using an implicator and a triangular norm based on a fuzzy covering of a universe of discourse. Feng et al. [17] studied the reduction of a fuzzy covering and the fusion of multi-fuzzy covering systems based on evidence theory and rough set theory. Ma [35] defined two new types of fuzzy covering rough set models, which can be regarded as bridges that link covering rough set theory and fuzzy rough set theory.

Atanassov's intuitionistic fuzzy (IF) [2,3] set theory is regarded as an intuitively straightforward extension of fuzzy set theory. This theory has been successfully applied in many fields for decision analysis and pattern recognition [9,10,52,54,59,60]. Rough sets and IF sets (IFSs) both capture particular facets of the same notion-imprecision. Studies on the combination of IFS theory and rough set theory are being acknowledged as a positive approach to rough set theory. For example, Zhou et al. [70,72] examined IF rough set approximation operators using both constructive and axiomatic approaches. The characterizations of IF rough set approximations were studied in [71]. Zhang [69] researched generalized IF rough sets based on IF coverings. Two pairs of generalized lower and upper IF rough approximation operators were constructed using an IF covering, an IF triangular norm, and an IF implicator. Zhang et al. [66] provided a systematic study of a general framework of IF rough sets. Huang et al. [22,23] discussed dominance-based IF and interval-valued IF rough set models and their applications.

One important characteristic of various rough set models is that a target concept is always characterized by the upper and lower approximations under a single granulation, that is, the concept is depicted by the available knowledge induced from a single relation on the universe. However, as illustrated in [43], in many cases describing the target concept through multiple relations on the universe according to user requirements or problem-solving targets is more reasonable. Qian et al. [43] extended Pawlak's single-granulation rough set model to a multi-granulation rough set (MGRS) model to widely apply rough set theory in practical applications. In Qian et al.'s MGRS model, two different basic models were defined; one was the optimistic MGRS, and the other was the pessimistic MGRS [40]. Rapid developments are presently being achieved in the MGRS area. For instance, Qian et al. [38] extended the rough set model based on a tolerance relation to an incomplete rough set model based on multi-granulations, in which set approximations were defined through multiple tolerance relations on the universe. Abu-Donia [1] studied rough approximations through a multi-knowledge base. She and He [47] discussed the fundamental properties of the MGRS model. Lin et al. [29,30] developed neighborhood-based and covering-based MGRSs to extend the theory of MGRS. Liang et al. [26] proposed an efficient rough feature selection algorithm for large-scale data sets inspired by multi-granulation. Yang et al. [63] generalized the MGRS model into a fuzzy environment. Huang et al. [20,21] examined multi-granulation IF approximation, IF rough sets, and their matrix-based reduction methods.

Based on the research progress described above, many important results have been achieved in the domains of covering-based rough sets, IF rough sets, and MGRSs; however, a few studies have been conducted on their combination. In some sense, without considering the applications, there are three basic problems on the combination of covering-based rough sets, IF rough sets, and MGRSs: (1) covering-based IF rough set: how to generalize the covering-based fuzzy rough set to covering-based IF rough set, (2) multi-granulation covering-based IF rough set: how to address covering-based IF rough set in the viewpoint of multi-granulation, and (3) rule acquisition: how to acquire simpler rules on the basis of multi-granulation covering-based IF rough set. However, to the best of our knowledge, there did not exist a unified framework for these three developments, which is efficient for studying covering-based IF rough sets. This concept is the driving force for our research. In the present paper, we propose the IF graded covering rough set in the viewpoint of fuzzy graded coverings [33], construct a multi-granulation IF rough set based on IF graded neighborhood induced by IF graded coverings, and study the matrix-based reduction method for the presented graded-neighborhood-based multi-granulation IF rough set by generalizing the multi-granulation IF rough set based on IF relations [21]. Table 1 describes the connections of some related papers in details.

The rest of this paper is organized as follows: Section 2 briefly introduces the preliminary concepts considered in this study. Section 3 presents IF graded covering and IF graded neighborhood and discusses the uncertainty measures for the IF graded neighborhood approximation space induced by an IF graded covering approximation space. Section 4 proposes a novel IF rough set based on IF graded neighborhood, discusses some of the basic properties and rough and precision degrees of this IF rough set, and introduces a matrix-based computation method for the upper and lower approximations of this IF rough set model. Section 5 presents the concept of multi-granulation IF graded approximation space, in which we define the distance between two IF graded neighborhood system sets to differentiate them. We also define the optimistic and pessimistic multi-granulation IF rough sets in the multi-granulation IF graded neighborhood approximation space and discuss their basic properties. Section 6 presents our analysis of the reducts and matrix-based computation methods of the multi-granulation IF graded neighborhood approximation space presented in Section 5. Section 7 discusses further possible generalizations related to IF rough set based on the IF graded neighborhood presented in this study. Finally, Section 8 concludes this paper.

2. Basic concepts

This section reviews some of the basic definitions of covering rough sets, fuzzy β-covering rough sets, IFSs, IF rough sets, MGRSs, and multi-granulation IF rough sets.

Definition 2.1. [74] Let U be a universe of discourse, and $C=\{K|K\subseteq U\}$ be a family of subsets of U. If no element of C is empty, and $\bigcup_{K\in C}K=U$, then C is called a covering of U. We call the ordered pair (U, C) a covering approximation space.

Definition 2.2. [75] Let (U, C) be a covering approximation space. Then $N_C(x)=\bigcap\{K|K\in C\wedge x\in K\}$ is called the neighborhood of $x \in U$ with respect to (U, C).

Definition 2.3. [76] Let (U, C) be a covering approximation space, and $X\subseteq U$. The lower approximation $\underline{P}_C(X)$ and the upper approximation $\overline{P}_C(X)$ of X are defined as $\underline{P}_C(X)=\{x|x\in U\wedge N_C(x)\subseteq X\}$ and $\overline{P}_C(X)=\{x|x\in U\wedge N_C(x)\cap X\neq\phi\}$, respectively.

Definition 2.4. [7] Let (U, C) be a covering approximation space, and $X\subseteq U$. $\underline{P}_C(X)$ and $\overline{P}_C(X)$ are the lower and upper approximations of X, respectively. The rough degree $r_C(X)$ and the precision degree $p_C(X)$ of X with respect to (U, C) are defined as follows: $r_C(X)=\frac{|\underline{P}_C(X)|}{|\overline{P}_C(X)|}$, $p_C(X)=1-\frac{|\underline{P}_C(X)|}{|\overline{P}_C(X)|}$, where $|X|$ denotes the cardinality of set X and $|\overline{P}_C(X)|\neq 0$. Obviously, $0\leq r_C(X)\leq 1$, $0\leq p_C(X)\leq 1$.

By introducing the concepts of fuzzy β-covering and fuzzy β-neighborhood, Ma [31] defined a new type of fuzzy covering rough set models.

Definition 2.5. [35] Let U be an arbitrary universal set, and $F(U)$ be the fuzzy power set of U. For $\beta\in(0, 1]$, we call $\widetilde{C}=\{\widetilde{C}_1,\widetilde{C}_2,\cdots,\widetilde{C}_m\}$, with $\widetilde{C}_i\in F(U)(i=1,2,\cdots,m)$, a fuzzy β-covering of U, if $\bigcup_{i=1}^{m}\widetilde{C}_i(x)\geq\beta$ for each $x\in U$. $(U,\widetilde{C})$ is called a fuzzy covering approximation space.