Knowledge-Based Systems 107 (2016) 261–270

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

Optimal targeting of nonlinear chaotic systems using a novel evolutionary computing strategy

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ARTICLE INFO

Article history:
Received 5 October 2015
Revised 7 June 2016
Accepted 13 June 2016
Available online 15 June 2016

Keywords:
Chaos control
Chaotic dynamics
Optimization
Computational intelligence
Teaching-learning-based optimization

ABSTRACT

Control of chaotic systems to given targets is a subject of substantial and well-developed research issue in nonlinear science, which can be formulated as a class of multi-modal constrained numerical optimization problem with multi-dimensional decision variables. This investigation elucidates the feasibility of applying a novel population-based metaheuristics labeled here as Teaching-learning-based optimization to direct the orbits of discrete chaotic dynamical systems towards the desired target region. Several consecutive control steps of small bounded perturbations are made in the Teaching-learning-based optimization strategy to direct the chaotic series towards the optimal neighborhood of the desired target rapidly, where a conventional controller is effective for chaos control. Working with the dynamics of the well-known Hénon as well as Ushio discrete chaotic systems, we assess the effectiveness and efficiency of the Teaching-learning-based optimization based optimal control technique, meanwhile the impacts of the core parameters on performances are also discussed. Furthermore, possible engineering applications of directing chaotic orbits are discussed.

1. Introduction

Chaos is a kind of characteristic of non-linear systems, which is a bounded unstable dynamic behavior that exhibits sensitive dependence on initial conditions and includes infinite unstable periodic motions. Although it appears to be stochastic, it occurs in a deterministic nonlinear system under deterministic conditions. In recently years, growing interests from physics, chemistry, biology, electronics, controls and instrumentation have stimulated the studies of chaos so as to improve the industrial and manufacturing systems and processes which exhibit chaotic phenomena [1]. Control of chaotic systems is one of important and well-developed research issues in nonlinear science [2–4], and it is possible that a minute perturbation of the control parameter could redirect chaos towards the desired region and stabilize it [5]. In the last decade, chaotic control has been shown to cover a wide spectrum of real world applications in engineering [6–17].

Since the pioneering work of Hübler's on chaos control in 1989 [18], a wide variety of approaches have been proposed for the appropriate control of chaotic systems. Most of the control techniques have been based on the *OGY* method [5,19]. OGY exploits the exponential sensitivity of chaotic systems by using minute perturbations to direct the system towards a desired target in a short time. By extending the work of Ott et al. [5], Grebogi and Lai [20] described a method that converts the motion on a chaotic attractor to a desired attracting time periodic motion by making only small time dependent perturbations of a control parameter. This allows for a more generic choice of the feedback matrix and implementation to higher-dimensional systems. *Optimal control theory based approach* is an alternative approach for the control of chaotic dynamic systems [21]. Paskota et al. [22] applied the optimal control theory to calculate an open-loop controller and direct the orbit of a chaotic system towards the neighborhood of the desired target. Abarbanel et al. [3] demonstrated the use of an explicit single-step control method for directing a nonlinear system to the target orbit and maintaining it there. On the other hand, a few research studies have dealt with the control and synchronization problems of chaotic systems using the *variable structure control* scheme. Yu [23] examined the variable structure control strategy for the control of chaos in dynamic systems. In their study, by switching between two configurations of a perturbed parameter in chaotic systems, sliding regions can be created in which the desired performance lies. The stabilization and tracking of a periodic signal of the Rössler system have also been studied. Wang and Su [24] proposed an adaptive complementary variable structure control for chaotic synchronization. Based on Lyapunov's stability theory and the Babalat's lemma the proposed controller has been shown to render

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http://dx.doi.org/10.1016/j.knosys.2016.06.019

the synchronous error to zero. Fuh and Tung [25] presented an effective approach for controlling chaos by using a *differential geometric method* which transformed nonlinear dynamics into linear form algebraically, so that linear control techniques can be used. It has been shown that the proposed method is not only able to control chaotic motion to a steady state but also to any desired periodic orbit. *Linear feedback methods* [26,27] and *nonlinear feedback control* [28–30] are feasible approaches to direct chaotic systems to a steady state. [31] addressed the control of discrete-time chaotic dynamic systems using conventional feedback control strategies. Further, Chen and Dong [32] presented how to use the canonical feedback controllers to control chaotic trajectory of a continuous-time nonlinear system in converging to its equilibrium points and, more significantly, to its multiperiodic orbits including unstable limit cycles. Chen and Han [28] stabilize the controlled system at origin and synchronize two Genesio systems by designing a nonlinear feedback controller, whose stability could be easily guaranteed by using Hurwitz stability analysis approach. Based on the *adaptive control approach* [33], Liao and Tsai [34] constructed an adaptive observer-based driven system to synchronize the drive system whose dynamics are subjected to the system's disturbances and/or some unknown parameters. By appropriately selecting the observer gains, the synchronization and stability of the overall systems can be guaranteed with the Lyapunov approach. Fotsin and Bowong [35] addressed the problem of control and synchronization of coupled second-order oscillators. Firstly, they designed feedback controller to stabilize the system at its equilibrium. Then an adaptive observer was designed to synchronize the states of the master and slave oscillators using a single scalar signal corresponding to an observable state variable of the driving oscillator. Aghababa and Hashtarkhani [36] addressed the issue of synchronizing two different uncertain chaotic systems with unknown and different bounds via adaptive control method. Nijmeijer and Mareels [37] reformulated the chaotic synchronization as an *observer design problem*. Yang and Chen [38] provided some new observer-based criteria for discrete-time generalized chaos synchronization. Bai and Lonngren [39] utilized *active control theory* to synchronize a coupled Lorenz system. Tang and Wang [40] proposed an adaptive active control approach to make the states of two identical Chua's systems with unknown constant parameters to be asymptotically synchronized. Lu and Zhang [41] proposed the *backstepping design technique* for controlling Chen's chaotic attractor based on parameters identification. Wu and Lu [42] first designed an observer to identify the unknown parameter of Lü system, then applied the backstepping approach to control the uncertain Lü system to bounded points. Xu and Teo [43] considered the asymptotical stabilization problem of discrete chaotic systems based on the *impulsive control scheme*. By means of the Lyapunov stability theory and algebraic inequality techniques, sufficient conditions for asymptotical stability of the impulsive controlled discrete systems were obtained. Based on the impulsive control approach, Kemih et al. [44] addressed the satellite attitude control problem subjected to deterministic external perturbations which induced chaotic motions. Theorems on the stability of impulsive control systems were developed to find the conditions under which the chaotic systems can be asymptotically controlled to the origin by using impulsive control. As for the fuzzy approach [45], Poursamad and Davaie-Markazi [46] presented a robust *adaptive fuzzy control algorithm* for controlling unknown chaotic systems. The fuzzy system is designed to mimic an ideal controller, based on sliding-mode control. The robust controller is designed to compensate for the difference between the fuzzy controller and the ideal controller. The adaptive laws are derived in the Lyapunov sense to guarantee the stability of the controlled system. In addition to the above methods, some researches applied *optimization based methods* to direct chaos to targeted regions. From the viewpoint of optimization, control of chaotic systems could be formulated as multi-modal constrained numerical optimization problems [47–49]. Genetic algorithm [50], simplex-annealing strategy [51], Particle swarm optimization [52], and Differential Evolution [53] have been considered. Wang et al. [51] proposed an effective hybrid optimization strategy by combining the probabilistic jump search of simulated annealing with the convex polyhedron-based geometry search of Nelder-Mead Simplex method. The hybrid optimization strategy was applied to direct orbits of chaotic systems to a desired target region and to synchronize the two chaotic systems. Simulations results obtained on Hénon Map demonstrated the effectiveness of their hybrid approach.

In the past two decades, population-based optimization has attracted great attention from both academia and industry in many fields not limited in system science [54–62]. Recently, a new population-based metaheuristics, labeled as the Teaching-learning-based optimization (TLBO), has been proposed [63–67] as an alternative to genetic algorithm (GA) [68], particle swarm optimization (PSO) [69,70] and Differential Evolution (DE) [71] for continuous optimization problems. The TLBO is inspired by the process of the teaching process and learning process of students in a class. In TLBO, firstly a population of solutions which is composed of teacher and students is initialized randomly, in which the most knowledgeable individual with the best fitness value is generally regarded as the teacher, while the remaining individuals in population are considered as students. Then the population is evolved to find optimal solutions through *teaching phase* in which the teacher helps the students to improve their grades as well as the *learning phase* in which the students improve their grades through interactions among themselves. Compared with GA, PSO and DE, TLBO has some attractive characteristics. It uses simple differential operation between teacher and students to create new candidate solutions, as well as to guide the search toward the most promising region. The conventional TLBO only contains one adjustable controlling parameter which facilitates easy tuning and implementation, while in GA, PSO and DE more parameters need to be set in appropriate manner so as to guarantee the searching performance. Nowadays, TLBO has attracted much attention and wide applications in different fields since its birth in 2011 [72,73]. Application areas cover dynamic economic emission dispatch [74], structural optimization [75], power system [76], heat exchangers [77,78], thermoelectric cooler [79], chaotic time series prediction [80], planning and scheduling [81–85], bioinformatics [86] and engineering optimization problems [87–95] etc., which demonstrate the effectiveness and efficiency of the TLBO based algorithms.

To date, there has been a lack of research study on TLBO for chaos control. The objective of this investigation is explicitly set out to fulfill this role. In this study, the TLBO is applied to direct the orbits of chaotic dynamical systems, which could be formulated as multimodal numerical optimization problems with high dimensions. Simulations results based on Hénon Map and Ushio Map are then obtained to verify the effectiveness and efficiency of TLBO, and the effects of some core parameters are also investigated.

2. Problem formulation

Consider the following discrete chaotic dynamic system:

$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k)), \quad k = 1, 2, \ldots, N \tag{1}$$

where state $\mathbf{x}(k) \in R^n$, and $\mathbf{f}$: $R^n \rightarrow R^n$ is continuously differentiable.

To direct the system towards a desired target, often minute perturbation $\mathbf{u}(k) \in R^n$ is added to the chaotic system. The system depicted in Eq. (1) can then be reformulated as follows:

$$\mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k)) + \mathbf{u}(k), k = 0, 1, \cdots, N-1 \tag{2}$$